

# **Grammatical inference and subregular phonology**

Adam Jardine  
Rutgers University

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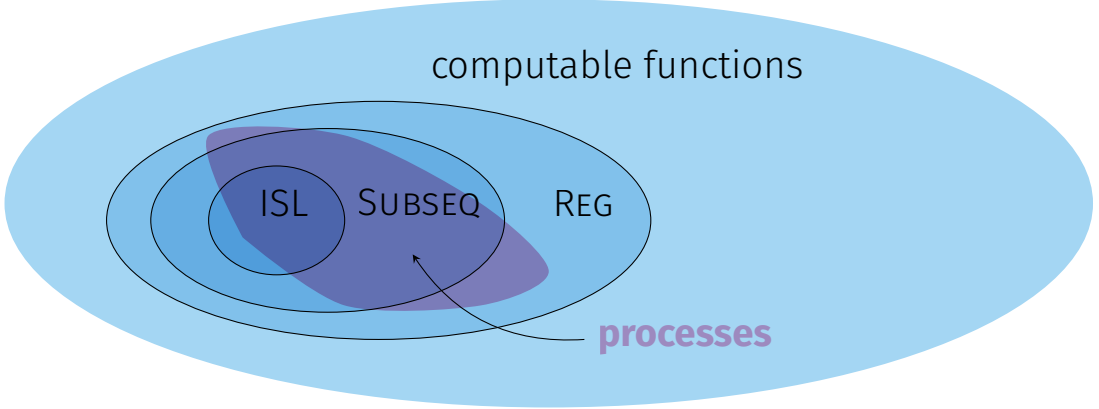
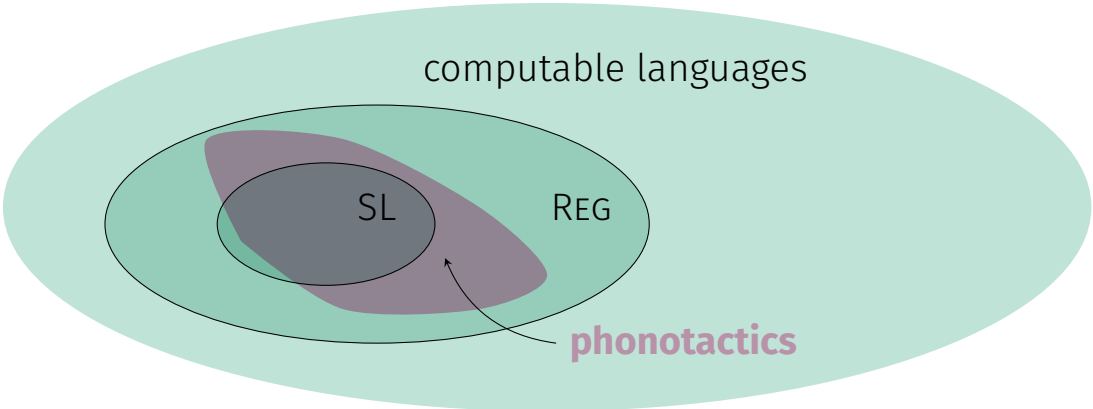
# Review

“[V]arious formal and substantive universals are intrinsic properties of the language-acquisition system, these providing a schema that is applied to data and that determines in a highly restricted way the general form and, in part, even the substantive features of the grammar that may emerge upon presentation of appropriate data.”

Chomsky, *Aspects*

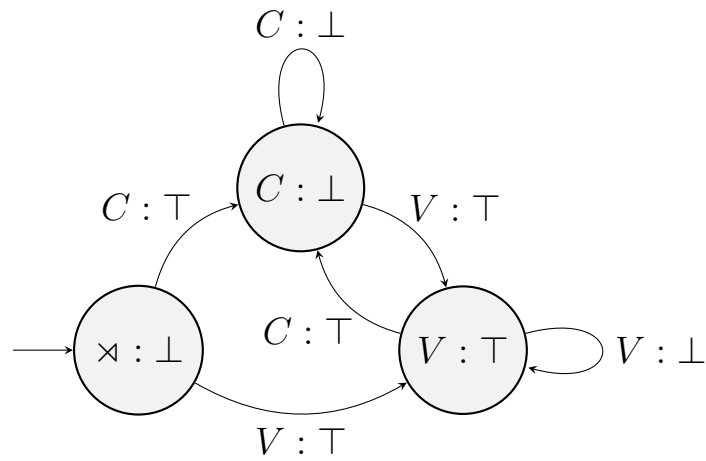
“[I]f an algorithm performs well on a certain class of problems then it necessarily pays for that with degraded performance on the set of all remaining problems.”

Wolpert and Macready (1997), *NFL Thms.*



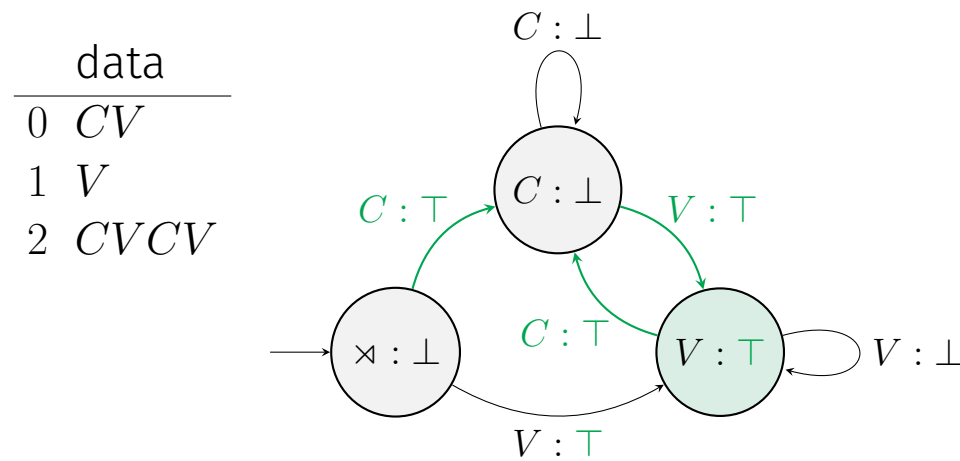
# Review

- Computational characterizations of phonological patterns identify **structure** that can be used by a learner



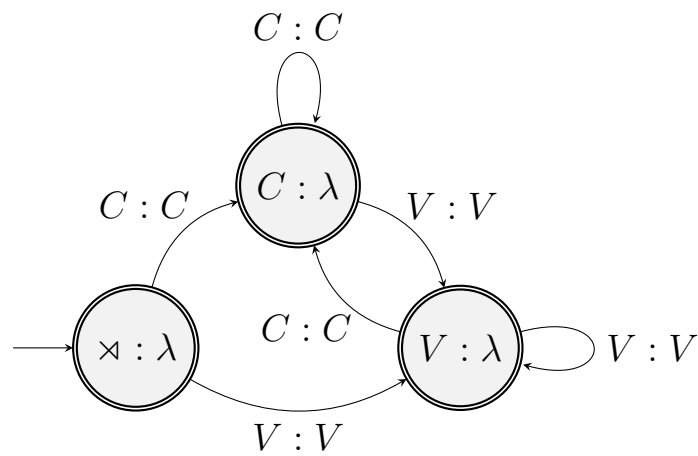
# Review

- Computational characterizations of phonological patterns identify **structure** that can be used by a learner



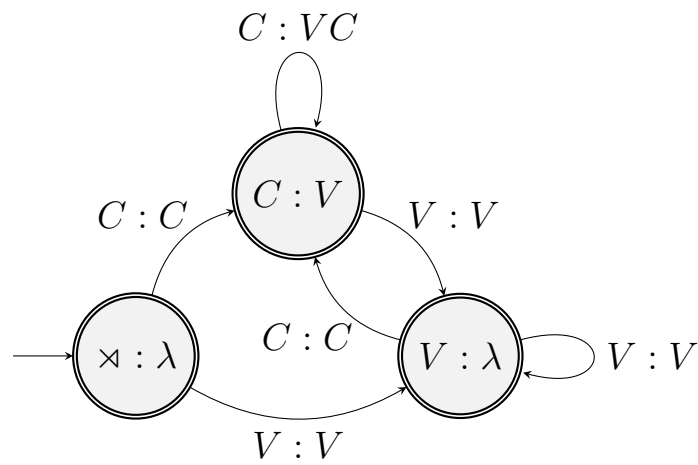
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# Review

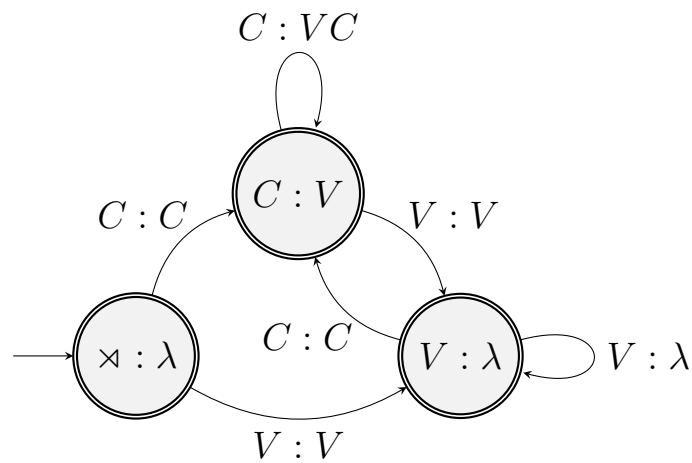
- Computational characterizations of phonological patterns identify **structure** that can be used by a learner





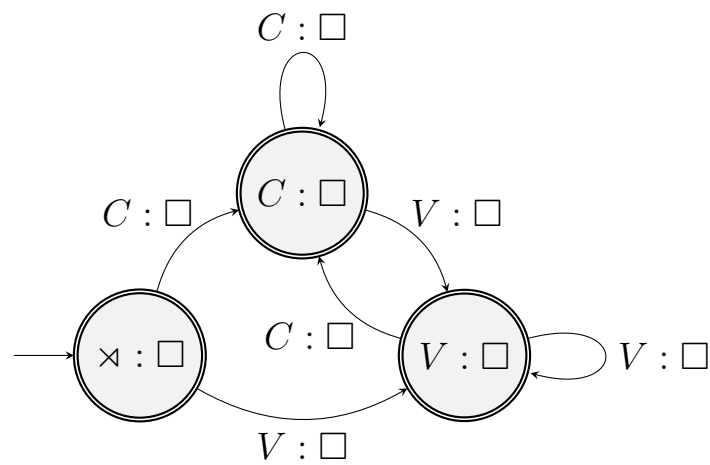
# Review

- Computational characterizations of phonological patterns identify **structure** that can be used by a learner



# Review

- Computational characterizations of phonological patterns identify **structure** that can be used by a learner



# Today

- Using automata structure for learning
  - ISL functions
  - **SL distributions**
- Open questions

# Learning ISL functions

# Learning input strictly local functions

- When learning languages, presentation is a sequence of examples of  $L$

$t$	datum
0	$V$
1	$CVCV$
2	$CVVCVCV$
$\vdots$	

- When learning **functions**, ...

# Learning input strictly local functions

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$t$	datum
0	$V$
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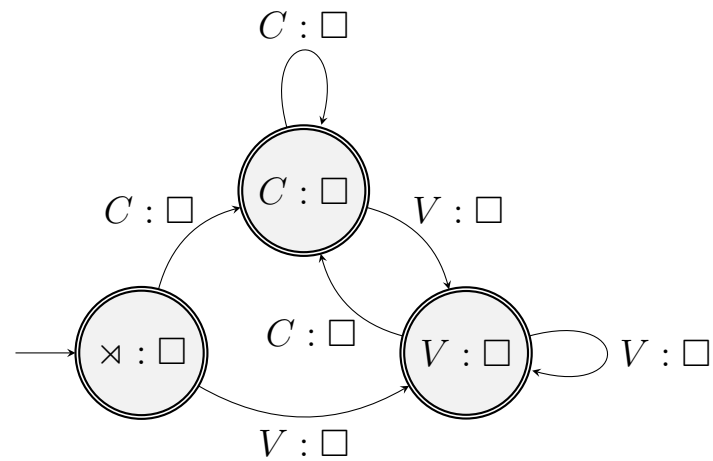
- When learning **functions**, presentation is of example **pairs** from  $f$

$t$	datum
0	$(C, CV)$
1	$(CVC, CVCV)$
2	$(CVCV, CVCV)$
$\vdots$	

# Learning input strictly local functions

$t$	datum
0	$(C, CV)$
1	$(CVC, CVCV)$
2	$(CVCV, CVCV)$
3	$(VCVC, VCVCV)$

→ ?



# Learning input strictly local functions

- The **longest common prefix (lcp)** is the longest initial sequence shared by a set of strings

$$lcp(\{CVCV, CVCCV, CVCVC\}) =$$



# Learning input strictly local functions

- The **longest common prefix (lcp)** is the longest initial sequence shared by a set of strings

$$\text{lcp}(\{CVCV, CVCCV, CVCVC\}) = CVC$$

$$\text{lcp}(\{CVCV, CCVCV, CVCVC\}) =$$

# Learning input strictly local functions

- The **longest common prefix (lcp)** is the longest initial sequence shared by a set of strings

$$\text{lcp}(\{CVCV, CVCCV, CVCVC\}) = CVC$$

$$\text{lcp}(\{CVCV, CCVCV, CVCVC\}) = C$$

# Learning input strictly local functions

- The **longest common prefix (lcp)** is the longest initial sequence shared by a set of strings

$$\text{lcp}(\{CVCV, CVCCV, CVCVC\}) = CVC$$

$$\text{lcp}(\{CVCV, CCVCV, CVCVC\}) = C$$

- Call our data sequence  $d \subset f$

$(CV, CV)$

$(CVC, CVC)$

$(CVCVC, CVCVC)$

$(VCVVC, VCVVC)$

$(VCVV, VCV)$

$$d^p(w) = \text{lcp}(d(w\Sigma^*))$$

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$(CVCVC, CVCVC)$

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$$d^p(w) = \text{lcp}(d(w\Sigma^*))$$

$$d^p(CVC) = \dots$$

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$$d^p(VCVV) = \dots$$

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- Call our data sequence  $d \subset f$

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$$d^p(w) = \text{lcp}(d(w\Sigma^*))$$

$(CVC, CVC)$

$$d^p(CVC) = CVC$$

$(CVCVC, CVCVC)$

$$d^p(VCVV) = VCV$$

$(VCVVC, VCVVC)$

$(VCVV, VCV)$

# Learning input strictly local functions

- Call our data sequence  $d \subset f$

$(CV, CV)$

$(CVC, CVC)$

$(CVCVC, CVCVC)$

$(VCVVC, VCVVC)$

$(VCVV, VCV)$

$$d^p(w) = \mathbf{1cp}(d(w\Sigma^*))$$

$$d^p(CVC) = CVC$$

$$d^p(VCVV) = VCV$$

$$d_w(u) = d^p(w)^{-1}d(wu)$$

# Learning input strictly local functions

- Call our data sequence  $d \subset f$

$$(CV, CV)$$

$$(CVC, CVC)$$

$$(CVCVC, CVCVC)$$

$$(VCVVC, VCVVC)$$

$$(VCVV, VCV)$$

$$d^p(w) = \mathbf{1cp}(d(w\Sigma^*))$$

$$d^p(CVC) = CVC$$

$$d^p(VCVV) = VCV$$

$$d_w(u) = d^p(w)^{-1}d(wu)$$

$$\begin{aligned} d_{CV}(C) &= d^p(CV)^{-1}d(CVC) \\ &= (CV)^{-1}CVC = C \end{aligned}$$



# Learning input strictly local functions

- Call our data sequence  $d \subset f$

$$(CV, CV)$$

$$(CVC, CVC)$$

$$(CVCVC, CVCVC)$$

$$(VCVVC, VCVVC)$$

$$(VCVV, VCV)$$

$$d^p(w) = \mathbf{1cp}(d(w\Sigma^*))$$

$$d^p(CVC) = CVC$$

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$$\begin{aligned} d_{CV}(C) &= d^p(CV)^{-1}d(CVC) \\ &= (CV)^{-1}CVC = C \end{aligned}$$

$$\begin{aligned} d_{VCV}(V) &= d^p(VCV)^{-1}d(VCVV) \\ &= (VCV)^{-1}VCV = \lambda \end{aligned}$$

# Learning input strictly local functions

- Call our data sequence  $d \subset f$

$$(CV, CV)$$

$$(CVC, CVC)$$

$$(CVCVC, CVCVC)$$

$$(VCVVC, VCVVC)$$

$$(VCVV, VCV)$$

$$d^p(w) = \mathbf{1cp}(d(w\Sigma^*))$$

$$d^p(CVC) = CVC$$

$$d^p(VCVV) = VCV$$

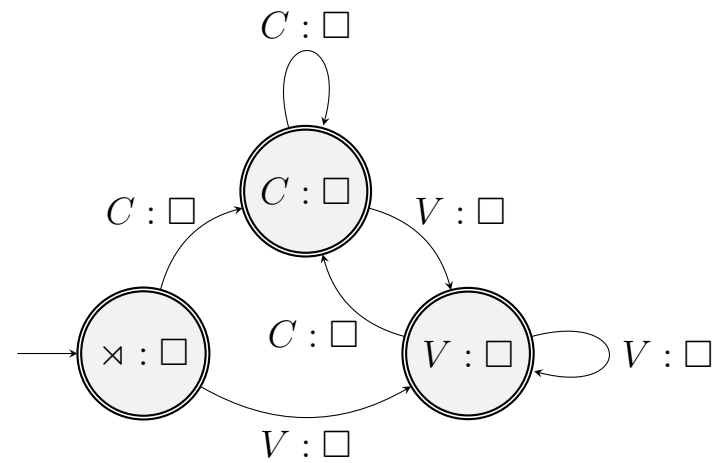
$$d_w(u) = d^p(w)^{-1}d(wu)$$

$$\begin{aligned} d_{CV}(C) &= d^p(CV)^{-1}d(CVC) \\ &= (CV)^{-1}CVC = C \end{aligned}$$

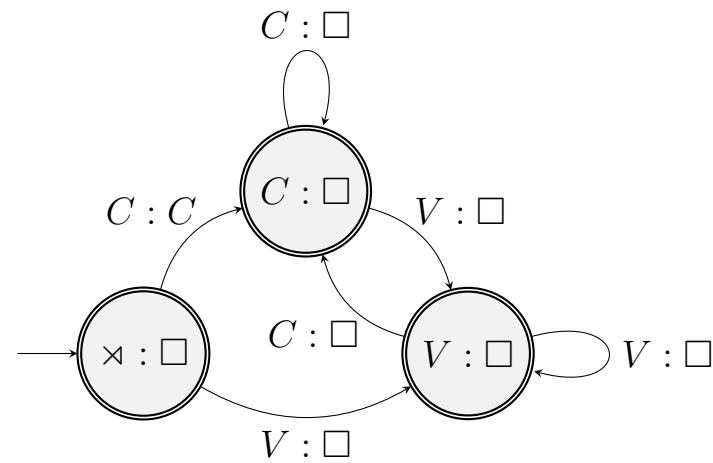
$$\begin{aligned} d_{VCV}(V) &= d^p(VCV)^{-1}d(VCVV) \\ &= (VCV)^{-1}VCV = \lambda \end{aligned}$$

$$d_w^p(u) = \mathbf{1cp}(d_w(u\Sigma^*))$$

*(CVC, CVC)*  
*(CVV, CV)*  
*(CVCCV, CVCCV)*  
*(CCVCC, CCVCC)*  
*(CCCVCV, CCCVCV)*  
*(CVVCV, CVVCV)*  
*(V, V)*

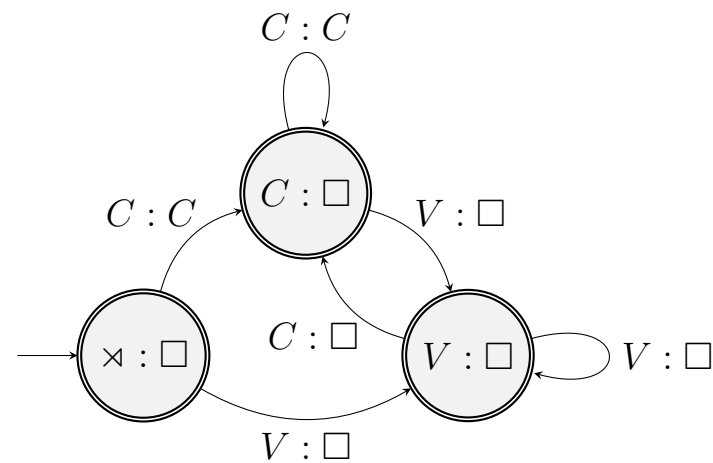


(CVC, CVC)  
 (CVV, CV)  
 (CVCCV, CVCCV)  
 (CCVCC, CCVCC)  
 (CCCVCV, CCCVCV)  
 (CVVCV, CVCV)  
 (V, V)



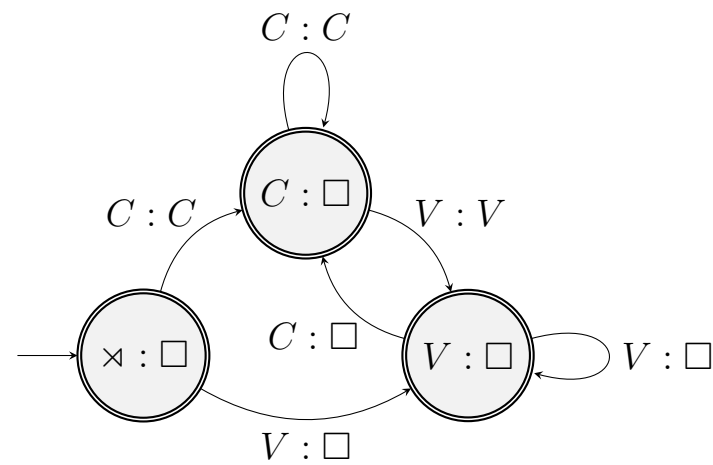
$$d_{\lambda}^p(C) = C$$

(CVC, CVC)  
 (CVV, CV)  
 (CVCCV, CVCCV)  
 (CCVCC, CCVCC)  
 (CCCVCCV, CCCVCCV)  
 (CVVCCV, CVCV)  
 (V, V)



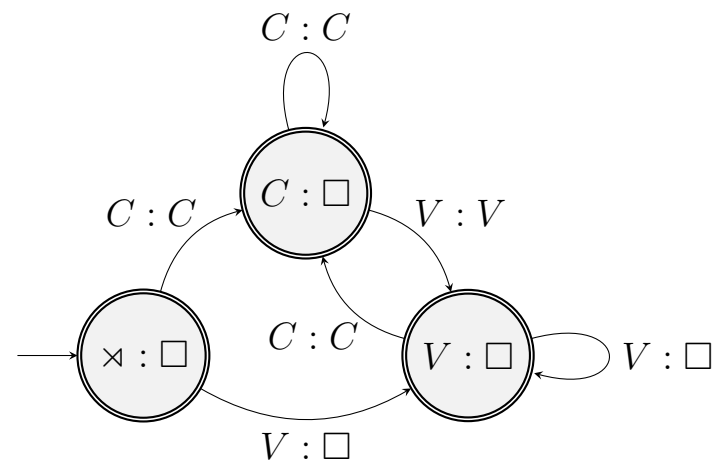
$$d_C^p(C) = C$$

(CVC, CVC)  
 (CVV, CV)  
 (CVCCV, CVCCV)  
 (CCVCC, CCVCC)  
 (CCCVCV, CCCVCV)  
 (CVVCV, CVVCV)  
 (V, V)



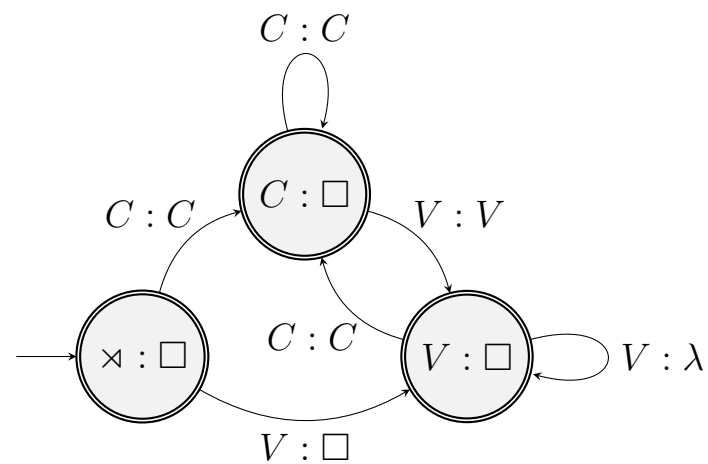
$$d_C^p(V) = V$$

(CVC, CVC)  
 (CVV, CV)  
 (CVCCV, CVCCV)  
 (CCVCC, CCVCC)  
 (CCCVCV, CCCVCV)  
 (CVVVCV, CVVVCV)  
 (V, V)



$$d_{CV}^p(C) = C$$

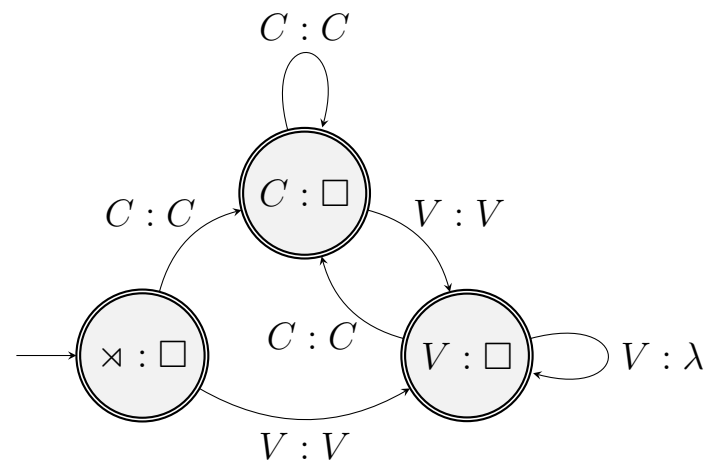
(CVC, CVC)  
 (CVV, CV)  
 (CVCCV, CVCCV)  
 (CCVCC, CCVCC)  
 (CCCVCV, CCCVCV)  
 (CVVCV, CVCV)  
 (V, V)



$$d_{CV}^p(V) = \lambda$$

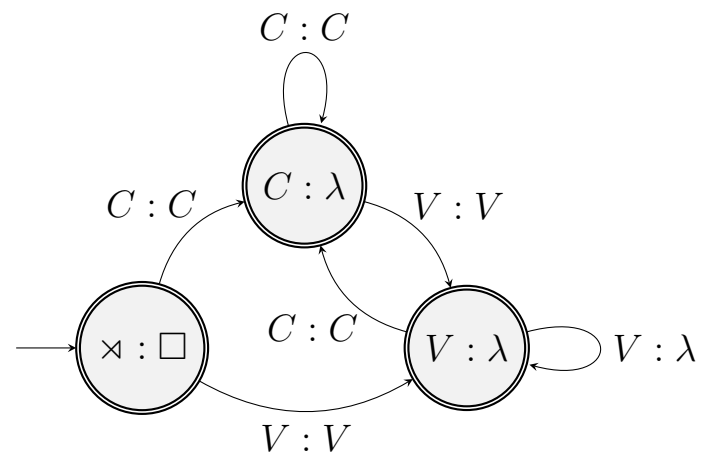


(CVC, CVC)  
 (CVV, CV)  
 (CVCCV, CVCCV)  
 (CCVCC, CCVCC)  
 (CCCVCV, CCCVCV)  
 (CVVCV, CVVCV)  
 (V, V)



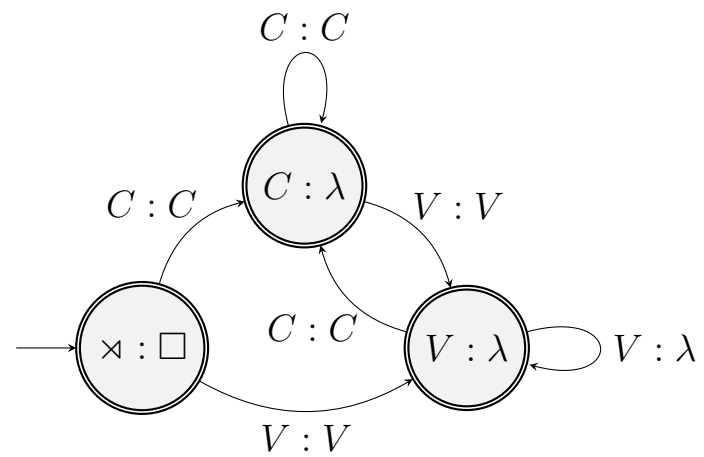
$$d_{\lambda}^p(V) = V$$

(CVC, CVC)  
 (CVV, CV)  
 (CVCCV, CVCCV)  
 (CCVCC, CCVCC)  
 (CCCVCV, CCCVCV)  
 (CVVCV, CVCV)  
 (V, V)



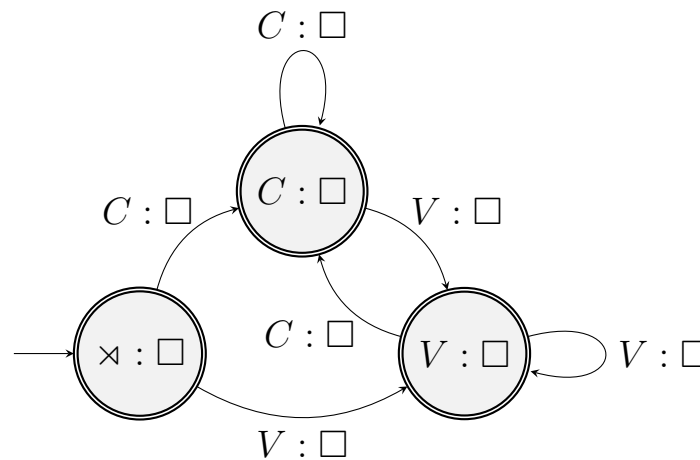
$$d^p(CVC)^{-1}d(CVC) = \lambda, \quad d^p(V)^{-1}d(V) = \lambda$$

(CVC, CVC)  
 (CVV, CV)  
 (CVCCV, CVCCV)  
 (CCVCC, CCVCC)  
 (CCCVCV, CCCVCV)  
 (CVVCV, CVCV)  
 (V, V)



# Learning input strictly local functions

- As any two  $ISL_k$  functions share the same structure, this method ILPD-learns the  $ISL_k$  functions



- This method extends to *any* class of functions that shares such a structure (Jardine et al., 2014)

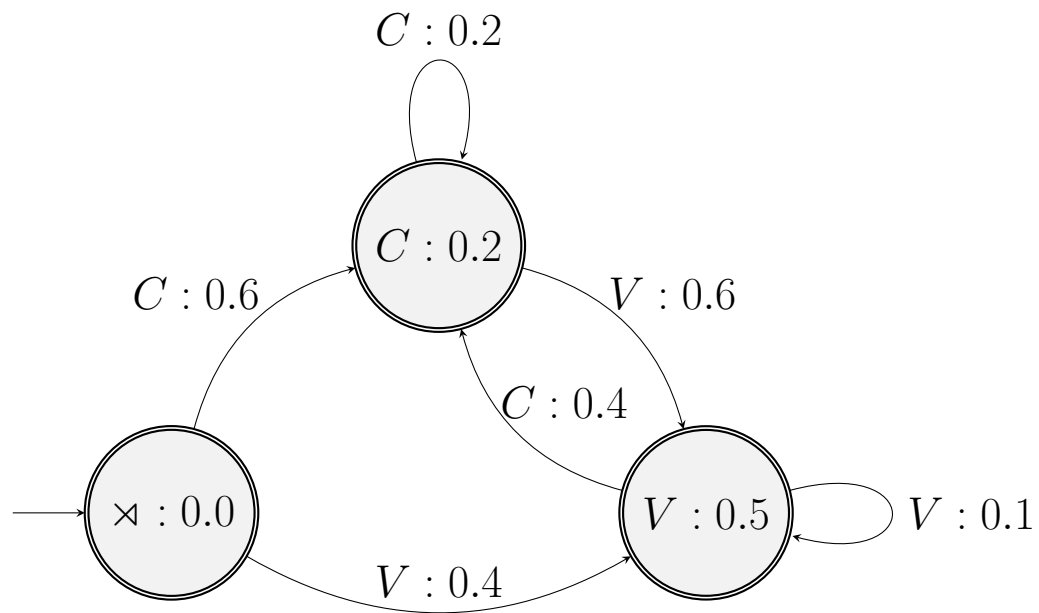
# Learning input strictly local functions

- A learning algorithm for grammars that explicitly encode computational properties of phonological patterns
- Learning for OSL ([Chandlee et al., 2015](#)) and tier-based OSL ([Burness and McMullin, 2019](#)) use a similar (yet distinct) method
- Learning URs uses this same structural concept (Hua et al. in progress)
- Learning for optional ISL processes uses the same basic idea (Heinz in progress) based on [Beros and de la Higuera \(2016\)](#)

# Learning SL distributions

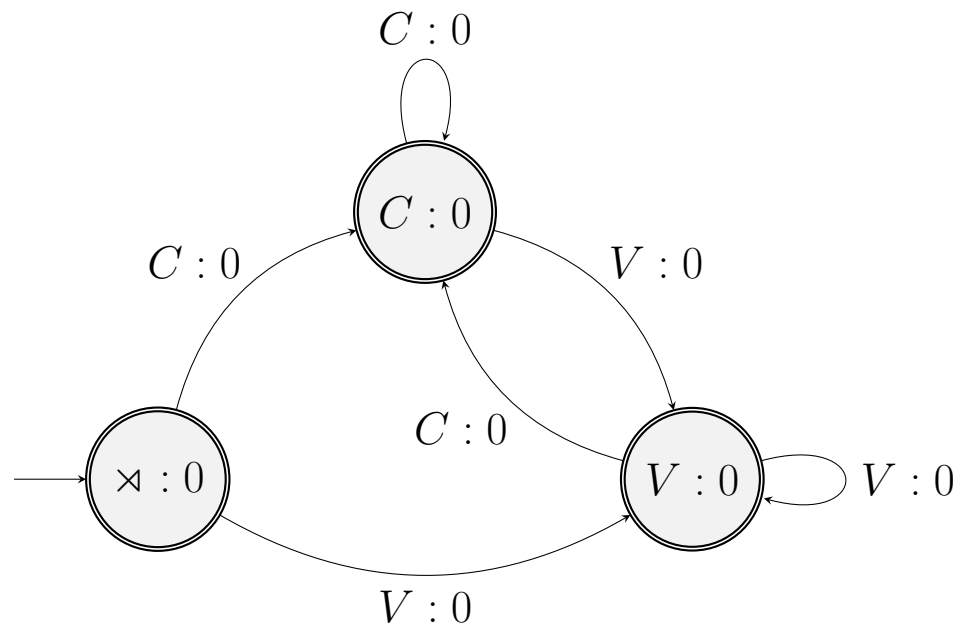
# Learning strictly local distributions

- Probability distributions can be described with **the same structure**.



# Learning strictly local distributions

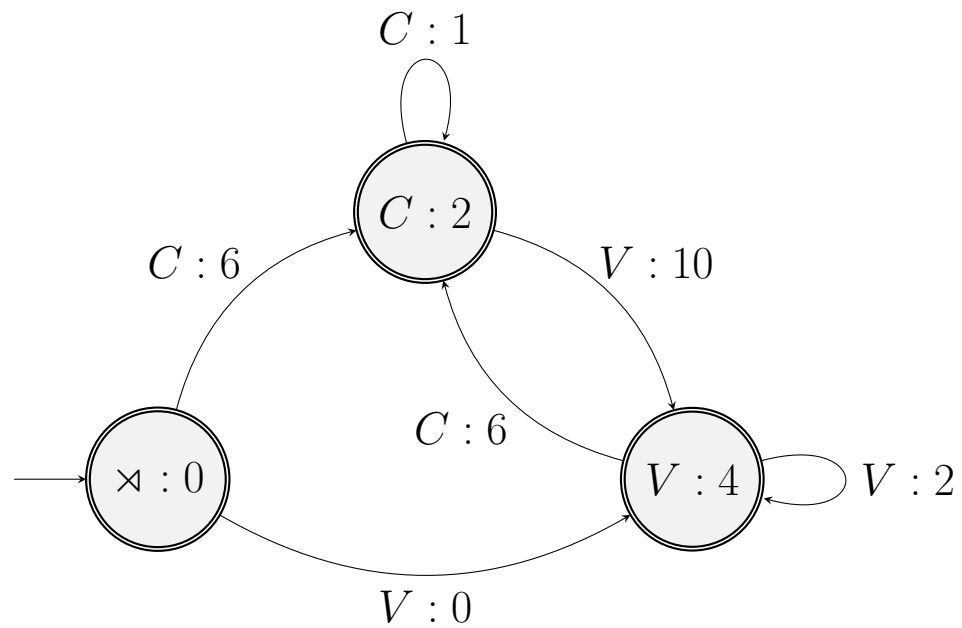
*CVC*  
*CVV*  
*CVCCV*  
*CVCVC*  
*CVCV*  
*CVVCV*





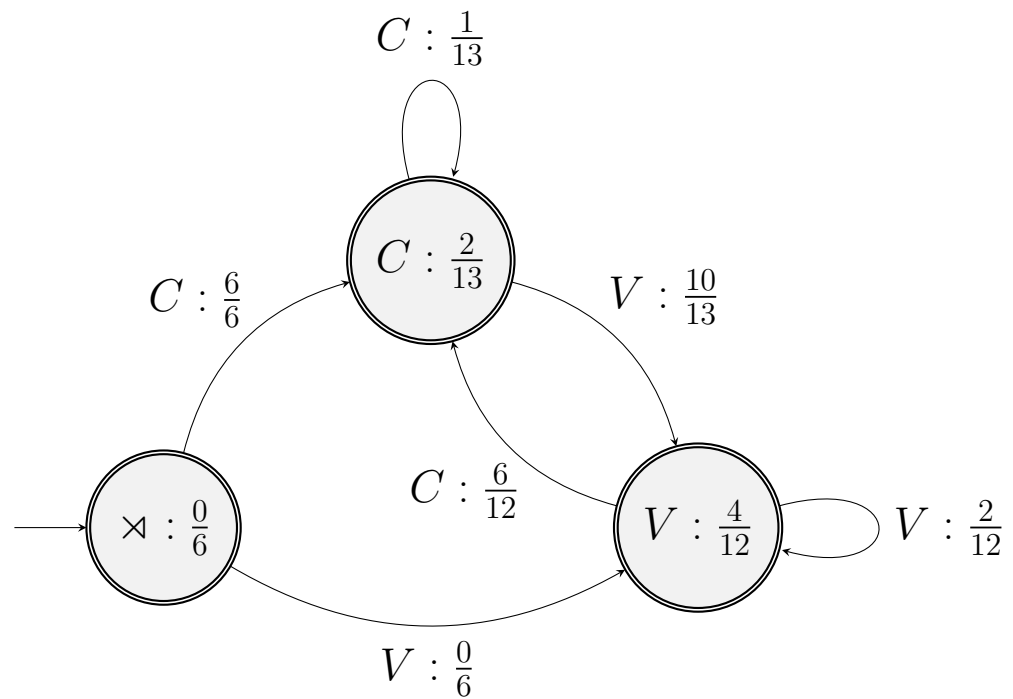
# Learning strictly local distributions

*CVC*  
*CVV*  
*CVCCV*  
*CVCVC*  
*CVCV*  
*CVVCV*



# Learning strictly local distributions

*CVC*  
*CVV*  
*CVCCV*  
*CVCVC*  
*CVCV*  
*CVVCV*



# Learning structured distributions

- This same technique can be extended to...
  - Learning strictly piecewise distributions: [Heinz and Rogers \(2010\)](#)
  - Learning SL distributions over features: [Heinz and Koirala \(2010\)](#)
  - ...

# Review

- Studying computational principles that underly phonological patterns identify structural properties for learning:
  - phonotactics
  - processes
  - stochastic generalizations
- A theory of phonology based on these principles derives typological predictions from learning

# Open questions

- Non-string representations are best characterized using **logic**  
(Jardine, 2016; Strother-Garcia, 2017)
- Learning with logic is a wide-open question  
(Strother-Garcia et al., 2016)
- Learning using features  
(Chandlee et al., 2019)
- Learning URs (Hua et al., in progress)
- Learning optionality (Heinz et al., in progress) and stochastic processes  
(wide open)
- Distinguishing accidental versus systematic gaps (Rawski in progress)

# Open questions

- A useful tool:

<https://github.com/alenaks/SigmaPie>