On the Logical Complexity of Autosegmental Representations

Adam Jardine

RUTGERS

Dept. of Linguistics, Rutgers University

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Introduction

- *Autosegmental representations* (ARs) are two-dimensional representations of phonological information

[félàmà] ‘junction’
(Mende; Leben, 1973)

- Two results in this paper:
  - *Tone mapping* is not *MSO-definable*, and thus categorically more complex than other phonological processes
  - ARs are *FO-definable* from strings, and thus are not dramatically more expressive than strings w.r.t. well-formedness
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Background

- What is the character of phonological generalizations?
  - **Well-formedness**
    blick vs. **bnick* (Chomsky and Halle, 1965)
  - **Processes**

write	/raɪt/	→[raɪt]
writer	/raɪt+ər/	→[raɪɾər]

- How do we best characterize cross-linguistic variation in well-formedness patterns and processes?

Background

- The **computational** character of phonology is (sub-)*Regular*:
  - **Well-formedness:** sub-classes of the **Regular sets** (Heinz and Idsardi, 2011, 2013; Rogers et al., 2013; McMullin and Hansson, 2016)
  - **Processes:** sub-classes of the **Regular relations** (Johnson, 1972; Kaplan and Kay, 1994; Heinz and Lai, 2013; Chandlee, 2014)

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- The **sub-Regular hypothesis for phonology** is a strong statement of the cognitive complexity and acquisition of phonology (Heinz, 2010; Rogers and Pullum, 2011; Rogers et al., 2013; Lai, 2015; McMullin and Hansson, 2015)

Background

- This hypothesis is in terms of *strings*
- Phonology has long been characterized with *non-string* structures like ARs (Goldsmith, 1976; Clements, 1976, *inter alia*)

- There can be no 'canonical' string encoding for ARs (Kornai, 1991, 1995)
- Modified finite-state machines of varying expressive power (Kay, 1987; Wiebe, 1992; Bird and Ellison, 1994; Kornai, 1991, 1995)

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- The Regular stringsets are exactly the **monadic second-order (MSO)-definable stringsets** (Büchi, 1960; Trakhtenbrot, 1961)
- The Regular string functions are properly included by **MSO-definable transductions** for strings
  (Engelfriet and Hoogeboom, 2001; Filiot and Reynier, 2016)

Background

- (Sub-)Regular hypothesis $\leftrightarrow$ **MSO-definable hypothesis**
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- We can directly compare AR processes to string processes

Tone mapping

Mende word tone (Leben, 1973; Goldsmith, 1976)

σ			$\sigma\sigma$			$\sigma\sigma\sigma$		
kɔ́	H	‘war’	pɛ́lɛ́	HH	‘house’	háwámá	HHH	‘waist’
mbû	F	‘owl’	ngílà	HL	‘dog’	félàmà	HLL	‘junction’
mbǎ	R	‘rice’	nìká	LH	‘cow’	ndàvúlá	LHH	‘sling’

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- Words choose among 5 melodies (*HLH)
- Plateaus of tone appear at the right edge of the word
  HHH, HLL
  *LLH, *HHL
- Contours appear at the right edge of the word
  R, LF, *RH

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*LLH, *HHL, *RH, *RHH, etc.

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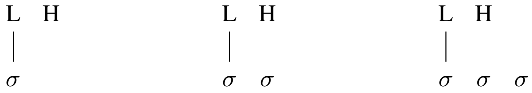

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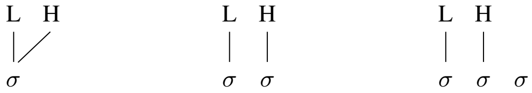

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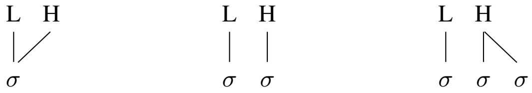

Tone mapping

- Some variation:
  - **Mende**: Start with first tone and first syllable, make pairs left-to-right
  - **Hausa**: Start with *last* tone and *last* syllable, make pairs *right-to-left* (Newman, 1986, 2000)
  - **Kikuyu**: Associate first tone to first *two* syllables, then make pairs left-to-right (Clements and Ford, 1979)
- **All**: Make pairs **one-by-one** until reaching some edge of the word

Logical transductions

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- Phonological processes are MSO-definable transductions
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Logical transductions

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- Phonological processes are MSO-definable transductions
- **Tone mapping is not MSO-definable**
- The following goes through:
  - Relational models and predicate logic
  - Logical transductions (Courcelle, 1994; Courcelle et al., 2012)
  - A proof of the claim

Logical transductions

Models

- Finite relational models

$$\langle U; R_1, R_2, ..., R_k \rangle$$

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- Strings over alphabet $\Sigma = \{a, b\}$:

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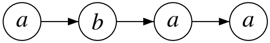
- Ex., *abaa* is

$$\langle \{1, 2, 3, 4\}_U; <, \{1, 3, 4\}_{P_a}, \{2\}_{P_b} \rangle$$



Logical transductions

Logics

- An **atomic predicate** $x = y$
- For each R_i of arity n, an atomic predicate $R_i(x_1, ..., x_n)$
- **First-order (FO)** logic defined recursively with connectives $\neg, \wedge, \vee, \rightarrow$ and quantifiers $\exists x$ and $\forall x$
- **Monadic second-order (MSO)** logic adds set quantifiers $\exists X, \forall X$ and unary set predicates $X(x)$

Logical transductions

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- **String** atomic predicates: $x = y$, $x < y$, $P_a(x)$, $P_b(x)$

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- Ex.,
  $(\forall x, y)[x < y \rightarrow \neg(P_b(x) \wedge P_a(y))] \wedge (\exists x, y)[P_a(x) \wedge P_b(y)]$

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- This describes the set of strings $a^n b^m$ for $n, m > 0$:

$$ab, aab, abb, aaab, aabb, abbb, aaaab, aaabb, aabbb, \ldots$$

Logical transductions

Logical transductions (Courcelle, 1994; Engelfriet and Hoogeboom, 2001)

$$\langle U; R_1, ..., R_k \rangle \to \langle V; S_1, ..., S_\ell \rangle$$

- **Interpretation** of output structures in logic of the input structures
  - φ_{dom} defining domain
  - A finite copy set C
  - For each S_i of arity n and $w \in C^n$, a formula $S_i^w(x_1, ..., x_n)$ in the logic of the input structure

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Logical transductions (Courcelle, 1994; Engelfriet and Hoogeboom, 2001)

$$\langle U; <, P_a, P_b \rangle \to \langle V; <', Q_a, Q_b \rangle$$

- Example: $\tau(a^n b \Sigma^m) \stackrel{\text{def}}{=} a^n b^{m+1}$ (ex. $\tau(abaa) = abbb$)

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- $C = \{1\}$ and

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\varphi_{\text{dom}} &\stackrel{\text{def}}{=} \varphi_{\text{string}} \\
<'(x,y) &\stackrel{\text{def}}{=} x < y \\
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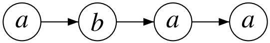
Input: 

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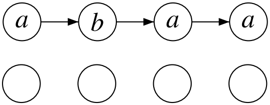

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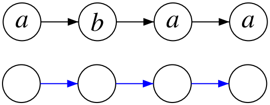

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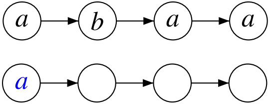

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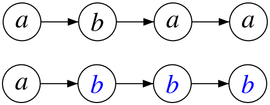

Logical transductions

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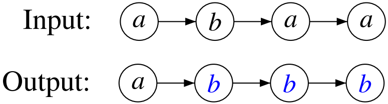
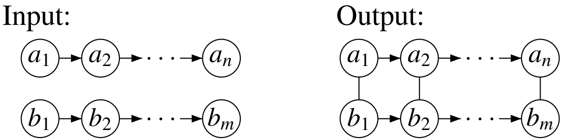


- Restatements of output structure in logic of the input structure
- MSO transductions are closed under composition (Courcelle, 1994)

Tone mapping is not MSO-definable

- (Mende) tone mapping is the following transduction:



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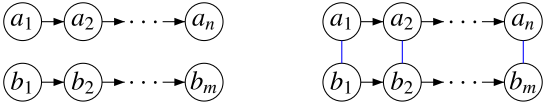

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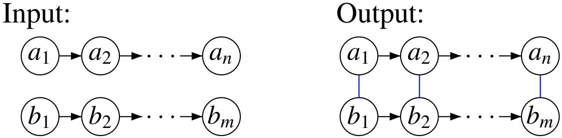

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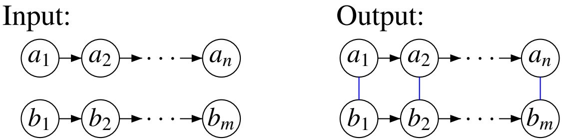

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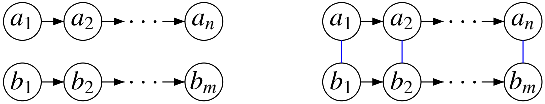

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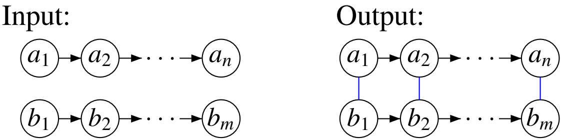

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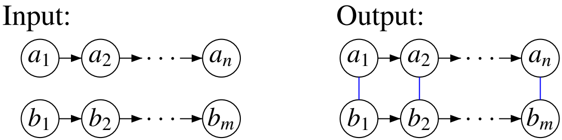

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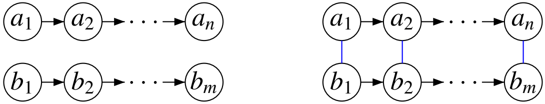

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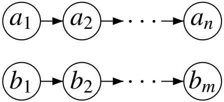
    - If $n < m$, for $n \leq i \leq m$, $(a_n, b_i) \in A$
    - If $n > m$, for $m \leq i \leq n$, $(a_i, b_m) \in A$
    - If $n = m$, $(a_n, b_m) \in A$
    - *Nothing else*

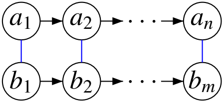
Tone mapping is not MSO-definable

- (Mende) tone mapping is the following transduction:

Input:



Output:



  - $\langle U; <, P_a, P_b \rangle \to \langle V; <', A, Q_a, Q_b \rangle$
  - Where A is the reflexive closure of:
    - For each $i < n, m$, $(a_i, b_i) \in A$
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  - **Theorem:** this transduction is not MSO-definable

Tone mapping is not MSO-definable

- Proof is by contradiction, comes in two parts

Tone mapping is not MSO-definable

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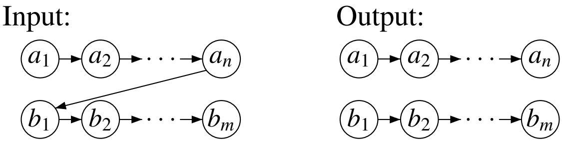
- **Part 1:** The following is MSO-definable:



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- **Part 2:** If tone mapping is MSO-definable, then the following is MSO-definable:

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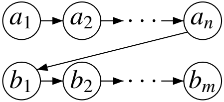
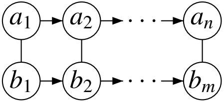


Output:



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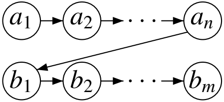
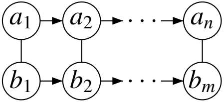


Output:



  - $\varphi_{\text{dom}} \stackrel{\text{def}}{=} \varphi_{a^n b^m}$
  - $A(x, y) \stackrel{\text{def}}{=} \varphi_A(x, y)$

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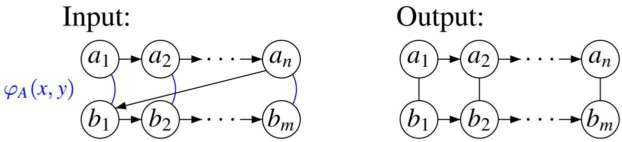

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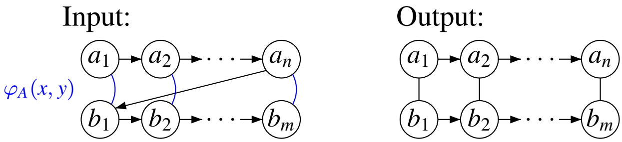

  - $\varphi_{\text{dom}} \stackrel{\text{def}}{=} \varphi_{a^n b^m}$
  - $A(x, y) \stackrel{\text{def}}{=} \varphi_A(x, y)$
- $(\forall x \exists y)[\varphi_A(x, y)] \wedge (\forall x, y, z)[(\varphi_A(x, y) \wedge \varphi_A(x, z)) \rightarrow y = z]$

Tone mapping is not MSO-definable

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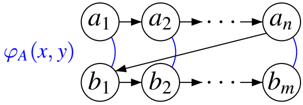
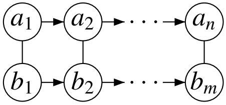


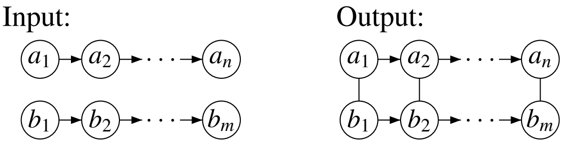
Output:



  - $\varphi_{\text{dom}} \stackrel{\text{def}}{=} \varphi_{a^n b^m}$
  - $A(x, y) \stackrel{\text{def}}{=} \varphi_A(x, y)$
- $(\forall x \exists y)[\varphi_A(x, y)] \wedge (\forall x, y, z)[(\varphi_A(x, y) \wedge \varphi_A(x, z)) \rightarrow y = z]$
  ↳This describes $a^n b^n$, **which is not MSO-definable**

Tone mapping is not MSO-definable

- The following transduction is not MSO-definable:



- Because MSO transductions are closed under composition, it can't be broken down into a finite number of MSO-definable steps
- This makes tone mapping more complex than other phonological processes, which are (at most) MSO-definable

Discussion

Interpreting the result

- Is tone different? (Hyman, 2011; Jardine, 2016a)

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- Is tone mapping the wrong generalization? (Dwyer, 1978; Shih and Inkelas, 2014)
- Are tone melodies finite? (Yli-Jyrä, 2013)

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Discussion

Interpreting the result

- Perhaps 'one-by-one' quality of mapping is *universal*
- This property is shared by all tone-mapping patterns
- Variation in realization of tone mapping patterns is extremely restricted (Jardine, 2016b, 2017)
- A full study of complexity of autosegmental tone processes is an important goal for future work

Discussion

The other result

- ARs **are** FO-definable from strings
- In terms of **well-formedness**, FO-statements over ARs are equivalent to FO-statements over strings
- Virtually all phonological well-formedness constraints are sub-FO (Graf, 2010b; Rogers et al., 2013)

Conclusion

- We used logical transductions to directly compare an AR processes to string processes
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Conclusion

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- Tone mapping is not MSO-definable, in contrast to all other phonological processes
- This negative result can be understood to be about language universals: one-by-one mapping is universal, and not subject to cross-linguistic variation
- Logical transductions are a powerful way to study phonological representation

Acknowledgments

Thanks to Jane Chandlee, Jeff Heinz, Thomas Graf, Jim Rogers, and three reviewers.

References I

Bird, S. and Ellison, T. M. (1994). One-level phonology: Autosegmental representations and rules as finite automata. *Computational Linguistics*, 20.

Bird, S. and Klein, E. (1990). Phonological events. *Journal of Linguistics*, 26:33–56.

Büchi, J. R. (1960). Weak second-order arithmetic and finite automata. *Zeitschrift für Mathematische Logik und Grundlagen der Mathmatik*, 6:66—92.

Chandlee, J. (2014). *Strictly Local Phonological Processes*. PhD thesis, University of Delaware.

Chandlee, J. and Lindell, S. (forthcoming). A logical characterization of strictly local functions. In Heinz, J., editor, *Doing Computational Phonology*. OUP.

Chomsky, N. and Halle, M. (1965). Some controversial questions in phonological theory. *Journal of Linguistics*, 1(2):pp. 97–138.

References II

Clements, G. N. (1976). *Vowel Harmony in Nonlinear Generative Phonology: An Autosegmental Model*. Bloomington: Indiana University Linguistics Club Publications.

Clements, G. N. and Ford, K. C. (1979). Kikuyu tone shift and its synchronic consequences. *Linguistic Inquiry*, 10:179–210.

Courcelle, B. (1994). Monadic second-order definable graph transductions: a survey. *Theoretical Computer Science*, 126:53–75.

Courcelle, B., Engelfriet, J., and Nivat, M. (2012). *Graph structure and monadic second-order logic: A language-theoretic approach*. Cambridge University Press.

Dwyer, D. (1978). What sort of tone language is Mende? *Studies in African Linguistics*, 9:167–208.

Engelfriet, J. and Hoogeboom, H. J. (2001). MSO definable string transductions and two-way finite-state transducers. *ACM Transations on Computational Logic*, 2:216–254.

Filiot, E. and Reynier, P. (2016). Transducers, logic, and algebra for functions of finite words. *ACM SIGLOG News*, 3(3):4–19.

References III

Goldsmith, J. (1976). *Autosegmental Phonology*. PhD thesis, Massachussets Institute of Technology.

Graf, T. (2010a). Comparing incomparable frameworks: A model theoretic approach to phonology. *University of Pennsylvania Working Papers in Linguistics*, 16(2):Article 10.

Graf, T. (2010b). Logics of phonological reasoning. Master's thesis, UCLA.

Heinz, J. (2010). Learning long-distance phonotactics. *LI*, 41:623–661.

Heinz, J., editor (forthcoming). *Doing Computational Phonology*. Oxford: Oxford University Press.

Heinz, J. and Idsardi, W. (2011). Sentence and word complexity. *Science*, 333(6040):295–297.

Heinz, J. and Idsardi, W. (2013). What complexity differences reveal about domains in language. *Topics in Cognitive Science*, 5(1):111–131.

Heinz, J. and Lai, R. (2013). Vowel harmony and subsequentiality. In Kornai, A. and Kuhlmann, M., editors, *Proceedings of the 13th Meeting on Mathematics of Language*, Sofia, Bulgaria.

References IV

Hyman, L. (2011). Tone: Is it different? In Goldsmith, J. A., Riggle, J., and Yu, A. C. L., editors, *The Blackwell Handbook of Phonological Theory*, pages 197–238. Wiley-Blackwell.

Jardine, A. (2014). Logic and the generative power of Autosegmental Phonology. In Kingston, J., Moore-Cantwell, C., Pater, J., and Staubs, R., editors, *Supplemental proceedings of the 2013 Meeting on Phonology (UMass Amherst)*, Proceedings of the Annual Meetings on Phonology. LSA.

Jardine, A. (2016a). Computationally, tone is different. *Phonology*, 33:247–283.

Jardine, A. (2016b). *Locality and non-linear representations in tonal phonology*. PhD thesis, University of Delaware.

Jardine, A. (2017). The local nature of tone association patterns. *Phonology*, 34:385–405.

Johnson, C. D. (1972). *Formal aspects of phonological description*. Mouton.

Kaplan, R. and Kay, M. (1994). Regular models of phonological rule systems. *Computational Linguistics*, 20:331–78.

References V

Kay, M. (1987). Nonconcatenative finite-state morphology. In *Proceedings, Third Meeting of the European Chapter of the Association for Computational Linguistics*, pages 2–10.

Kornai, A. (1991). *Formal Phonology*. PhD thesis, Stanford University.

Kornai, A. (1995). *Formal Phonology*. Garland Publication.

Lai, R. (2015). Learnable versus unlearnable harmony patterns. *LI*, 46:425–451.

Leben, W. R. (1973). *Suprasegmental phonology*. PhD thesis, Massachussets Institute of Technology.

McMullin, K. and Hansson, G. O. (2015). Locality in long-distance phonotactics: evidence for modular learning. In *NELS 44, Amherst, MA*. in press.

McMullin, K. and Hansson, G. O. (2016). Long-distance phonotactics as Tier-Based Strictly 2-Local languages. In *Proceedings of AMP 2015*.

Newman, P. (1986). Tone and affixation in Hausa. *Studies in African Linguistics*, 17(3).

References VI

Newman, P. (2000). *The Hausa Language: An encyclopedic reference grammar*. New Haven: Yale University Press.

Rogers, J., Heinz, J., Fero, M., Hurst, J., Lambert, D., and Wibel, S. (2013). Cognitive and sub-regular complexity. In *Formal Grammar*, volume 8036 of *Lecture Notes in Computer Science*, pages 90–108. Springer.

Rogers, J. and Pullum, G. (2011). Aural pattern recognition experiments and the subregular hierarchy. *Journal of Logic, Language and Information*, 20:329–342.

Shih, S. and Inkelas, S. (2014). A subsegmental correspondence approach to contour tone (dis)harmony patterns. In Kingston, J., Moore-Cantwell, C., Pater, J., and Staubs, R., editors, *Proceedings of the 2013 Meeting on Phonology (UMass Amherst)*, Proceedings of the Annual Meetings on Phonology. LSA.

Trakhtenbrot, B. A. (1961). Finite automata and logic of monadic predicates. *Doklady Akademii Nauk SSSR*, 140:326–329.

Wiebe, B. (1992). Modelling autosegmental phonology with multi-tape finite state transducers. Master's thesis, Simon Fraser University.

References VII

Yli-Jyrä, A. (2013). On finite-state tonology with autosegmental representations. In *Proceedings of the 11th International Conference on Finite State Methods and Natural Language Processing*, pages 90–98. Association for Computational Linguistics.