A weak but flexible logic for modeling phonological processes

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March 23, 2026

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Overview

Overview

Boolean monadic recursive schemes (BMRS) [1]

- Restricted fragment of recursive programs[2]
- Capture the subsequential functions[3] on strings
- Useful for describing phonological processes[4]
- Current work focusing on learning[5]

[1] (Bhaskar et al. 2020)

[2] (Moschovakis 2019)

[3] (Schützenberger 1977; Mohri 1997)

[4] (Chandlee and Jardine 2021)

[5] (Yolyan 2025)

What is phonology?

Phonology as functions

English plural

"-z"		**"-s"**	
[piz]	‘peas’	[k^{h}læsps]	‘clasps’
[t^{h}oʊz]	‘toes’	[mɪts]	‘mitts’
[dɑlz]	‘dolls’	[bloʊks]	‘blokes’
[dɑgz]	‘dogs’	[k^{h}ɑfs]	‘coughs’
[læbz]	‘labs’		

Phonology as functions

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[læbz]	‘labs’		

Analysis:

- **Underlying representation (UR)** of plural: /-z/
- A **process** mapping URs to **surface representations (SRs)**: (/piz/,[piz]), (/dɑlz/,[dɑlz]), (/k^{h}læspz/,[k^{h}læsps]), (/mɪtz/,[mɪts]), …

Goals of Generative phonology

(/piz/,[piz]), (/dɑlz/,[dɑlz]), (/k^{h}læspz/,[k^{h}læsps]),
(/mɪtz/,[mɪts]), ...

- How do we capture phonological processes?
    - SPE[6]: [-sonorant] → [-voice] / [-voice] __
    - OT[7]: Agree in voicing ≫ Don't change voicing
- What is a *possible* phonological process?

[6] Sound Pattern of English (Chomsky and Halle 1968)

[7] Optimality Theory (Prince and Smolensky 1993)

Goals of Generative phonology

High-tone spreading in Shambaa (Odden 1982)

[ku-ʃunt^{h}-a]	‘to wash’	$(/\sigma\sigma\sigma/, [\sigma\sigma\sigma])$
[ku-ɣoʃo-a]	‘to do’	$(/\sigma\sigma\sigma\sigma/, [\sigma\sigma\sigma\sigma])$
[ku-tʃí-ʃúnt^{h}-a]	‘to wash it’	$(/\sigma\acute{\sigma}\sigma\sigma/, [\sigma\acute{\sigma}\acute{\sigma}\sigma])$
[ku-ví-ɣóʃó-a]	‘to do them’	$(/\sigma\acute{\sigma}\sigma\sigma\sigma/, [\sigma\acute{\sigma}\acute{\sigma}\acute{\sigma}\sigma])$
[ku-ɣoʃo-a-ɣoʃo-a]	‘to do repeatedly’	$(/\sigma\sigma\sigma\sigma\sigma\sigma\sigma/, [\sigma\sigma\sigma\sigma\sigma\sigma\sigma])$
[ku-tʃí-ɣóʃó-á-ɣóʃó-a]	‘to do it repeatedly’	$(/\sigma\acute{\sigma}\sigma\sigma\sigma\sigma\sigma\sigma/, [\sigma\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\sigma])$

Goals of Generative phonology

Long-distance consonant harmony in Kikongo

[sos-ila]	‘searched for’	(/sos-ila/,	[sos-ila])
[sakid-ila]	‘congratulate for’	(/sakid-ila/,	[sakid-ila])
[ku-toːt-ila]	‘to harvest for’	(/ku-toːt-ila/,	[ku-toːt-ila])
[ku-kin-ina]	‘to dance for’	(/ku-kin-ila/,	[ku-kin-ina])
[ku-dumuk-ina]	‘to jump for’	(/ku-dumuk-ila/,	[ku-dumuk-ina])
[ku-dumuk-is-ina]	‘to make jump for’	(/ku-dumuk-is-ila/,	[ku-dumuk-is-ina])

Goals of Generative phonology

- Why?
  - ✓ High tone spreads to second-to-last syllable
  - × High tone spreads to center syllable

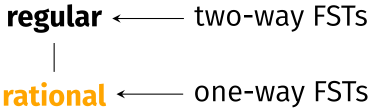
Phonology and computation

What kinds of functions are phonological processes?

- Heinz (2007);Heinz (2018): Phonological generalizations have a *computationally* restrictive character

What kinds of functions are phonological processes?

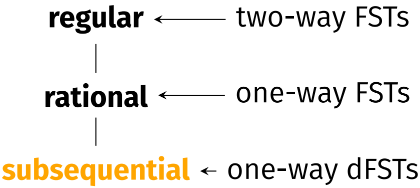
- Johnson (1972);Kaplan and Kay (1994): Phonological rules are **rational**



regular ← two-way FSTs

rational ← one-way FSTs

What kinds of functions are phonological processes?



regular ← two-way FSTs

rational ← one-way FSTs

subsequential ← one-way dFSTs

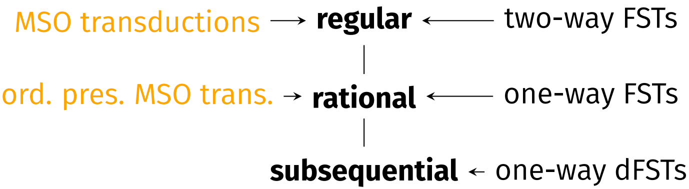
- Mohri (1997);Heinz and Lai (2013):
  Phonological processes are **subsequential**

What kinds of functions are phonological processes?

- Heinz (2018): Connections between logic and FLT[8] are useful for phonology
    - Ex: features, syllable structure, etc.

[8] Büchi (1960), McNaughton and Papert (1971), et. seq.

What kinds of functions are phonological processes?



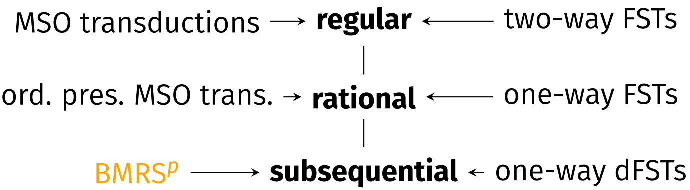
MSO transductions → **regular** ← two-way FSTs

ord. pres. MSO trans. → **rational** ← one-way FSTs

subsequential ← one-way dFSTs

Filiot and Reynier (2016)

What kinds of functions are phonological processes?



MSO transductions → **regular** ← two-way FSTs

ord. pres. MSO trans. → **rational** ← one-way FSTs

BMRS^p → **subsequential** ← one-way dFSTs

Bhaskar et al. (2020)

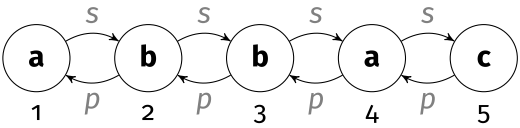
Boolean Monadic Recursive Schemes

BMRS

- Associate strings in Σ^* with structures of the form

$$\langle D; P_\sigma \text{ for } \sigma \in \Sigma, p, s \rangle$$

- E.g., *abbac* would be



where $P_a = \{1, 4\}$, $P_b = \{2, 3\}$, $P_c = \{c\}$.

BMRS

- $\mathtt{X}$ are index variables; $\mathtt{F}$ are function variables
- Syntax:

$$\begin{aligned} T \rightarrow \quad & \mathtt{x} \mid \top \mid \bot \mid \mathtt{f}(T) \mid s(T) \mid p(T) \mid \\ & \sigma(T) \mid \mathtt{if}\ T\ \mathtt{then}\ T\ \mathtt{else}\ T \end{aligned}$$

  where:

  - $\mathtt{x} \in \mathtt{X}$, $\mathtt{f} \in \mathtt{F}$, $\sigma \in \Sigma$
  - $\mathtt{x}$, $s(T)$ and $p(T)$ are index-typed, all others boolean
  - For $\sigma(T)$ and $\mathtt{f}(T)$, T must be index-typed
  - For $\mathtt{if}\ T\ \mathtt{then}\ T\ \mathtt{else}\ T$, each T is boolean-typed

BMRS

- Syntax:

$$\begin{aligned} T \to\ & \mathtt{x} \mid \top \mid \bot \mid \mathtt{f}(T) \mid s(T) \mid p(T) \mid \\ & \sigma(T) \mid \mathtt{if}\ T\ \mathtt{then}\ T\ \mathtt{else}\ T \end{aligned}$$

- Boolean predicates with one free variable, e.g.

$$\mathtt{if}\ a(\mathtt{x})\ \mathtt{then}\ b(p(\mathtt{x}))\ \mathtt{else}\ \bot$$

- Assume $\mathtt{f}(T)$ and $\sigma(T)$ return $\bot$ if T is undefined (at end or beginning of word)

BMRS

- A recursive, boolean, monadic **scheme** (or *program*) pairs $\vec{\mathtt{f}} = (\mathtt{f}_1, \ldots, \mathtt{f}_n)$ with definitions:

$$\begin{gathered}\mathtt{f}_1(\mathtt{x}) = T_1(\vec{\mathtt{f}}, \mathtt{x}) \\ \vdots \\ \mathtt{f}_n(\mathtt{x}) = T_n(\vec{\mathtt{f}}, \mathtt{x})\end{gathered}$$

  where each T_i is boolean.

- The semantics is given by least-fixed semantics of recursive programs

BMRS

An example

- Let $\Sigma = \{a, b, c\}$ and let $\vec{f} = (\mathtt{a}, \mathtt{b}, \mathtt{c})$.

$$\begin{aligned}
\mathtt{a}(\mathtt{x}) &= \mathtt{if}\ \mathtt{b}(\mathtt{x})\ \mathtt{then}\ \bot\ \mathtt{else}\ a(\mathtt{x}) \\
\mathtt{b}(\mathtt{x}) &= \mathtt{if}\ b(\mathtt{x})\ \mathtt{then}\ \top\ \mathtt{else} \\
&\qquad\quad \mathtt{if}\ a(\mathtt{x})\ \mathtt{then}\ \mathtt{b}(p(\mathtt{x}))\ \mathtt{else}\ \bot \\
\mathtt{c}(\mathtt{x}) &= c(\mathtt{x})
\end{aligned}$$

BMRS

An example

$$\mathtt{a}(\mathtt{x}) = \mathtt{if}\ \mathtt{b}(\mathtt{x})\ \mathtt{then}\ \bot\ \mathtt{else}\ a(\mathtt{x})$$

$$\begin{aligned}\mathtt{b}(\mathtt{x}) = {} & \mathtt{if}\ b(\mathtt{x})\ \mathtt{then}\ \top\ \mathtt{else} \\ & \quad \mathtt{if}\ a(\mathtt{x})\ \mathtt{then}\ \mathtt{b}(p(\mathtt{x}))\ \mathtt{else}\ \bot\end{aligned}$$

$$\mathtt{c}(\mathtt{x}) = c(\mathtt{x})$$

input: *a* *b* *a* *a* *c* *a*

BMRS

An example

$$\mathrm{a}(\mathtt{x}) = \mathtt{if}\ \mathrm{b}(\mathtt{x})\ \mathtt{then}\ \bot\ \mathtt{else}\ a(\mathtt{x})$$

$$\begin{aligned}\mathrm{b}(\mathtt{x}) = {} & \mathtt{if}\ b(\mathtt{x})\ \mathtt{then}\ \top\ \mathtt{else} \\ & \qquad \mathtt{if}\ a(\mathtt{x})\ \mathtt{then}\ \mathrm{b}(p(\mathtt{x}))\ \mathtt{else}\ \bot\end{aligned}$$

$$\mathrm{c}(\mathtt{x}) = c(\mathtt{x})$$

input:	*a*	*b*	*a*	*a*	*c*	*a*
$\mathrm{a}(\mathtt{x})$						
$\mathrm{b}(\mathtt{x})$						
$\mathrm{c}(\mathtt{x})$						

BMRS

An example

$$\texttt{a}(\texttt{x}) = \texttt{if}\ \texttt{b}(\texttt{x})\ \texttt{then}\ \bot\ \texttt{else}\ a(\texttt{x})$$

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$$\texttt{c}(\texttt{x}) = c(\texttt{x})$$

input:	*a*	*b*	*a*	*a*	*c*	*a*
$\texttt{a}(\texttt{x})$	$\top$					
$\texttt{b}(\texttt{x})$	$\bot$					
$\texttt{c}(\texttt{x})$	$\bot$					

BMRS

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$\mathtt{a}(\mathtt{x})$	$\top$	$\bot$				
$\mathtt{b}(\mathtt{x})$	$\bot$	$\top$				
$\mathtt{c}(\mathtt{x})$	$\bot$	$\bot$				

BMRS

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$$\mathrm{c}(\mathtt{x}) = c(\mathtt{x})$$

input:	*a*	*b*	*a*	*a*	*c*	*a*
a(x)	$\top$	$\bot$				
b(x)	$\bot$	$\top$	$\top$			
c(x)	$\bot$	$\bot$				

BMRS

An example

$$\mathtt{a}(\mathtt{x}) = \mathtt{if}\ \mathtt{b}(\mathtt{x})\ \mathtt{then}\ \bot\ \mathtt{else}\ a(\mathtt{x})$$

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$$\mathtt{c}(\mathtt{x}) = c(\mathtt{x})$$

input:	*a*	*b*	*a*	*a*	*c*	*a*
$\mathtt{a}(\mathtt{x})$	$\top$	$\bot$	$\bot$			
$\mathtt{b}(\mathtt{x})$	$\bot$	$\top$	$\top$			
$\mathtt{c}(\mathtt{x})$	$\bot$	$\bot$	$\bot$			

BMRS

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input:	*a*	*b*	*a*	*a*	*c*	*a*
$\mathtt{a}(\mathtt{x})$	$\top$	$\bot$	$\bot$	$\bot$		
$\mathtt{b}(\mathtt{x})$	$\bot$	$\top$	$\top$	$\top$		
$\mathtt{c}(\mathtt{x})$	$\bot$	$\bot$	$\bot$	$\bot$		

BMRS

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\mathtt{c}(\mathtt{x}) &= c(\mathtt{x})
\end{aligned}$$

input:	*a*	*b*	*a*	*a*	*c*	*a*
a(x)	⊤	⊥	⊥	⊥	⊥	⊤
b(x)	⊥	⊤	⊤	⊤	⊥	⊥
c(x)	⊥	⊥	⊥	⊥	⊤	⊥

BMRS

An example

$$\mathtt{a}(\mathtt{x}) = \texttt{if } \mathtt{b}(\mathtt{x}) \texttt{ then } \bot \texttt{ else } a(\mathtt{x})$$

$$\begin{aligned}\mathtt{b}(\mathtt{x}) = {} & \texttt{if } b(\mathtt{x}) \texttt{ then } \top \texttt{ else} \\ & \quad \texttt{if } a(\mathtt{x}) \texttt{ then } \mathtt{b}(p(\mathtt{x})) \texttt{ else } \bot\end{aligned}$$

$$\mathtt{c}(\mathtt{x}) = c(\mathtt{x})$$

input:	*a*	*b*	*a*	*a*	*c*	*a*
a(x)	⊤	⊥	⊥	⊥	⊥	⊤
b(x)	⊥	⊤	⊤	⊤	⊥	⊥
c(x)	⊥	⊥	⊥	⊥	⊤	⊥
output:	a	b	b	b	c	a

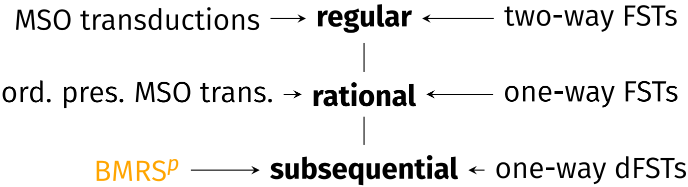
BMRS

- Fix an **output alphabet** Γ. We can include a function symbol in $\vec{\mathtt{f}}$ for each $\gamma \in \Gamma$.

$$
\begin{gathered}
\gamma_1(\mathtt{x}) = T_1(\vec{\mathtt{f}}, \mathtt{x}) \\
\vdots \\
\gamma_k(\mathtt{x}) = T_k(\vec{\mathtt{f}}, \mathtt{x}) \\
\mathtt{f}_1(\mathtt{x}) = T_{k+1}(\vec{\mathtt{f}}, \mathtt{x}) \\
\vdots \\
\mathtt{f}_n(\mathtt{x}) = T_{k+n}(\vec{\mathtt{f}}, \mathtt{x})
\end{gathered}
$$

- We can interpret this BMRS as a transduction from $\Sigma^* \rightarrow \Gamma^*$

BMRS - Expressivity



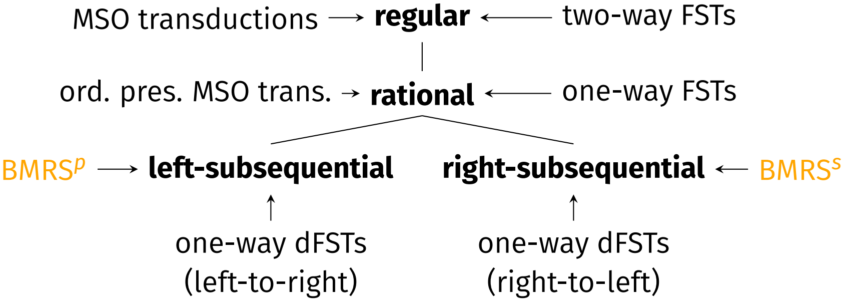
MSO transductions → **regular** ← two-way FSTs

ord. pres. MSO trans. → **rational** ← one-way FSTs

BMRS^p → **subsequential** ← one-way dFSTs

Bhaskar et al. (2020)

BMRS - Expressivity



MSO transductions → **regular** ← two-way FSTs

ord. pres. MSO trans. → **rational** ← one-way FSTs

BMRS^p → **left-subsequential**

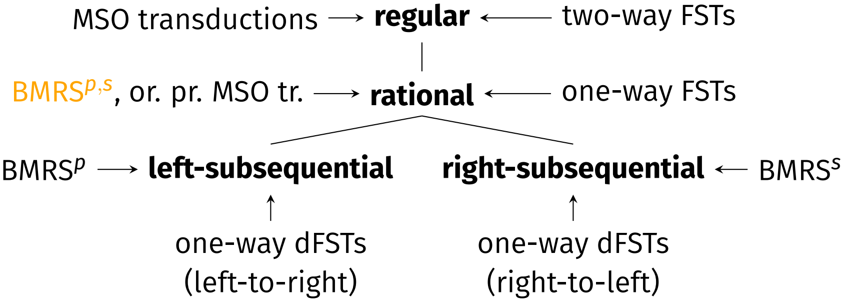
right-subsequential ← BMRS^s

one-way dFSTs (left-to-right)

one-way dFSTs (right-to-left)

Bhaskar et al. (2020)

BMRS - Expressivity



MSO transductions → **regular** ← two-way FSTs

BMRS^{p,s}, or. pr. MSO tr. → **rational** ← one-way FSTs

BMRS^{p} → **left-subsequential**

right-subsequential ← BMRS^{s}

one-way dFSTs (left-to-right) → **left-subsequential**

one-way dFSTs (right-to-left) → **right-subsequential**

Bhaskar and Jardine (in prep.)

BMRS and phonology

Shambaa spreading[9]

$$
\begin{gathered}
/\sigma\sigma\sigma\sigma\sigma/ \mapsto [\sigma\sigma\sigma\sigma\sigma] \\
/\sigma\acute{\sigma}\sigma\sigma\sigma/ \mapsto [\sigma\acute{\sigma}\acute{\sigma}\acute{\sigma}\sigma]
\end{gathered}
$$

$$
\begin{aligned}
\acute{\sigma}'(\mathtt{x}) &= \mathtt{if}\ \acute{\sigma}(\mathtt{x})\ \mathtt{then}\ \top\ \mathtt{else} \\
&\qquad\quad \mathtt{if}\ \mathtt{last}(\mathtt{x})\ \mathtt{then}\ \bot\ \mathtt{else}\ \acute{\sigma}'(p(\mathtt{x})) \\
\sigma'(\mathtt{x}) &= \mathtt{if}\ \acute{\sigma}'(\mathtt{x})\ \mathtt{then}\ \bot\ \mathtt{else}\ \top
\end{aligned}
$$

[9] We can include `last`(T) to input signature without increasing expressivity

BMRS and phonology

$$\begin{aligned}\acute{\sigma}'(\mathtt{x}) &= \mathtt{if}\ \acute{\sigma}(\mathtt{x})\ \mathtt{then}\ \top\ \mathtt{else}\\ &\qquad \mathtt{if}\ \mathtt{last}(\mathtt{x})\ \mathtt{then}\ \bot\ \mathtt{else}\ \acute{\sigma}'(p(\mathtt{x}))\\ \sigma'(\mathtt{x}) &= \mathtt{if}\ \acute{\sigma}'(\mathtt{x})\ \mathtt{then}\ \bot\ \mathtt{else}\ \top\end{aligned}$$

- Captures ’Elsewhere Condition” effects (Chandlee and Jardine 2021; Jardine and Oakden 2025)
- Even long-distance phonological processes are ‘myopic’ (Wilson 2003, 2006; though c.f. McCollum et al. 2020)

Learning

Learning

- dFSTs are effectively learnable (Oncina, García, and Vidal 1993)
- Yolyan (2025): fragments of BMRS^p are learnable

$$\begin{array}{ccc} a & b & \boxed{\begin{array}{c} a \\ \downarrow \\ b \end{array}} \end{array}$$

$$\begin{array}{cc} a & b \end{array}$$

To conclude

To conclude

- BMRS are a flexible, yet well-behaved model for computing string transductions
  - BMRS^p $\rightarrow$ left-subsequential
  - BMRS^s $\rightarrow$ right-subsequential
  - BMRS^{p,s} $\rightarrow$ rational
- They are useful for describing phonology
- There is a lot to learn about learning BMRS
- Also BMRS and B-RASP?

Thank you!

References I

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