Tone and the Generative Power of Autosegmental Phonology

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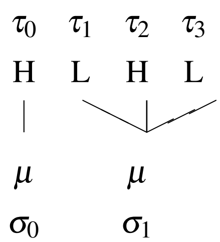
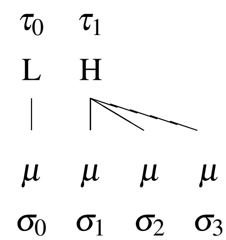
Adam Jardine, University of Delaware

ajardine@udel.edu https://sites.google.com/site/adamajardine/

The following details an approach to capturing Autosegmental Phonology (Goldsmith 1976) with word models and Monadic Second Order logic, in order to measure its computational complexity. Example autosegmental diagrams:



(1)



(2)

Word models: $\mathscr{W}_{\mathrm{T}} = \langle W_{\mathrm{T}}, \triangleleft^*, P_{\mathrm{T}} \rangle \qquad \mathscr{W}_{\Sigma} = \langle W_{\Sigma}, \triangleleft^*, P_{\Sigma} \rangle \qquad \mathscr{A} = \langle \mathscr{W}_{\mathrm{T}}, \mathscr{W}_{\Sigma}, \triangle \rangle$

Word models for (1):

$$\begin{aligned} \mathscr{W}_{\mathrm{T}} &= \big\langle \{0,1,2,3\}_{W_{\mathrm{T}}}, \{(0 \triangleleft^* 0), (0 \triangleleft^* 1), (0 \triangleleft^* 2), ..., (3 \triangleleft^* 3)\}_{\triangleleft^*}, \{0,2\}_{\mathrm{H}}, \{1,3\}_{\mathrm{L}} \big\rangle \\ \mathscr{W}_{\Sigma} &= \langle \{0,1\}_{W_{\Sigma}}, \{(0 \triangleleft^* 0), (0 \triangleleft^* 1), (1 \triangleleft^* 1)\}_{\triangleleft^*}, \{0,1\}_{\mu} \rangle \\ \mathscr{A} &= \big\langle \mathscr{W}_{\mathrm{T}}, \mathscr{W}_{\Sigma}, \big\{(0 \triangle 0), (1 \triangle 1), (2 \triangle 1), (3 \triangle 1)\big\}_{\triangle} \big\rangle \end{aligned} \tag{3}$$

Definitions and Axioms

- First and last in a set:

$$\mathrm{First}(X,x) \overset{\text{def}}{=} X(x) \wedge (\forall z)\,[X(z) \rightarrow (x \triangleleft^* z)] \tag{4}$$

$$\mathrm{Last}(X,x) \overset{\text{def}}{=} X(x) \wedge (\forall z)\,[X(z) \rightarrow (z \triangleleft^* x)] \tag{5}$$

- 'Less than' (irreflexive) precedence ($\triangleleft^+$):

$$x \triangleleft^+ y \overset{\text{def}}{=} (x \triangleleft^* y) \wedge (x \not\approx y) \tag{6}$$

- Immediate precedence ($\triangleleft$):

$$x \triangleleft y \overset{\text{def}}{=} \big(x \triangleleft^+ y\big) \wedge (\neg \exists z)\,\big[\big(x \triangleleft^+ z\big) \wedge \big(z \triangleleft^+ y\big)\big] \tag{7}$$

- General precedence of sets:

$$X \triangleleft^* Y \overset{\text{def}}{=} (\forall x,y)\,[X(x) \wedge Y(y) \rightarrow x \triangleleft^* y] \tag{8}$$

- Subsets:

$$X \subseteq Y \overset{\text{def}}{=} (\forall x)\,[X(x) \rightarrow Y(x)] \tag{9}$$

- A set of autosegments is X potentially associated with a TBU y:

$$\begin{aligned} \mathrm{Pot}(X,y) \quad \overset{\text{def}}{=} \quad & (X \subseteq W_{\mathrm{T}}) \wedge (y \in W_{\Sigma}) \wedge \\ & (\forall x)\,[X(x) \rightarrow x \triangle y] \end{aligned} \tag{10}$$

- Immediate precedence of sets:

$$X \triangleleft Y \overset{\text{def}}{=} (X \triangleleft^* Y) \wedge (\forall x,y)\,[(\mathrm{Last}(X,x) \wedge \mathrm{First}(Y,y)) \rightarrow x \triangleleft y] \tag{11}$$

- Contiguity of a set of variables:

$$\text{Contig}(X) \stackrel{\text{def}}{=} \exists(x,y)(\forall z)\left[(X(x) \wedge X(y)) \wedge ((x \triangleleft^{*} z) \wedge (z \triangleleft^{*} y) \leftrightarrow X(z))\right] \tag{12}$$

- Association of between a set of positions and another position is only true if the set is *contiguous* and *maximal*:

$$\begin{aligned} X \vartriangle y \quad \stackrel{\text{def}}{=} \quad & (\forall x)[X(x) \rightarrow (x \vartriangle y)] \wedge \\ & \text{Contig}(X) \wedge \\ & (\forall Z)\left[\text{Pot}(Z,y) \rightarrow (Z \subseteq X)\right] \end{aligned} \tag{13}$$

Axiom 1 *The no-crossing constraint.*

$$(\forall X,x,Y,y)\left[((X \vartriangle x) \wedge (Y \vartriangle y)) \rightarrow \big((X \triangleleft^{+} Y) \leftrightarrow (x \triangleleft^{+} y)\big)\right]$$

Axiom 2 *One-to-one association (either left to right or right to left).*

$$(\forall X,x)\left[\begin{array}{l} ((X \vartriangle x) \rightarrow (\forall y)\left[(y \triangleleft x) \rightarrow (\exists Y)\left[Y \vartriangle y\right]\right]) \vee \\ ((X \vartriangle x) \rightarrow (\forall y)\left[(x \triangleleft y) \rightarrow (\exists Y)\left[Y \vartriangle y\right]\right]) \end{array}\right]$$

Yield

We can then represent a linearized *yield* of the diagram by creating a new alphabet based on the associated sets of segments. The *general yield alphabet* Γ is the set of all possible associations:

Definition 1 *The* general yield alphabet *is* $\Gamma = \mathrm{T}^{*} \times \Sigma$

Lemma 1 *A* language-specific yield alphabet $\Gamma_L \subseteq \Gamma$ *of a language is finite iff there is some maximum bound on the length of strings in its left projection,* $\pi_0(\Gamma_L)$*:*

$$(\exists n \in \mathbb{N})\left[|\Gamma_L| = n\right] \iff (\exists m \in \mathbb{N})\left[max\left(\pi_0\left(\Gamma_L\right)\right) = m\right]$$

Axiom 3 *For any set of autosegmental diagrams in a language,* Γ_L *must be finite.*

Definition 2

$$\text{Yield}(\mathscr{A}) = \gamma_0\gamma_1...\gamma_n \in \Gamma_L^{*} \text{ where } \gamma_i = (w \in T^{*}, \sigma_i) \text{ and } w \vartriangle \sigma_i$$

Conjecture 1 *The set of strings represented by the yields of any set of autosegmental representations which at least follow Axioms 1,2, and 3 will be regular.*