

# Graph pattern learning for long-distance phonotactics

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# Introduction

- ▶ Languages have long-distance phonotactic patterns, especially in tone (Yip, 2002; Hyman, 2011)
- ▶ Dependencies between non-adjacent units can make the learning problem difficult (Hayes and Wilson, 2008)
- ▶ How are these patterns learned?

# Introduction

- ▶ Computationally local string learners form a strong theory of phonotactic learning, including many long-distance patterns (Heinz, 2009, 2010; Jardine and Heinz, 2016)
- ▶ Some tone patterns are beyond these learners

# Introduction

- ▶ These can be learned with a local **autosegmental** learner
- ▶ Idea: learn banned **subgraphs**
- ▶ Local autosegmental learning provides a strong theory of tone learning
- ▶ May be extended to long-distance segmental phonology as well

# Computational locality

- ▶ **Strictly Local (SL)** stringsets are those describable by a finite set of *banned substrings* (Rogers and Pullum, 2011)

# Computational locality

- ▶ Kagoshima Japanese

(Hirayama, 1951; Haraguchi, 1977; Kubozono, 2012)

a.	hána	'nose'	HL
b.	sakúra	'cherry blossom'	LHL
c.	kagaríbi	'watch fire'	LLHL
d.	kagaribí-ga	'watch fire' + NOM	LLLHL
			...
e.	haná	'flower'	LH
f.	usagí	'rabbit'	LLH
g.	kakimonó	'document'	LLLH
h.	kakimono-gá	'document' + NOM	LLLLH

...

## Computational locality

$$KJ = \{ \begin{array}{ll} \#HL\#, & \#LH\#, \\ \#LHL\#, & \#LLH\#, \\ \#LLHL\#, & \#LLLH\#, \\ \dots & \end{array} \}$$

- ▶  $G_{KJ} = \{\text{HLL}, \text{HH}, \text{HLH}, \text{LL}\#\}$

## Computational locality

$$KJ = \{ \begin{array}{ll} \#HL\#, & \#LH\#, \\ \#LHL\#, & \#LLH\#, \\ \#LLHL\#, & \#LLLH\#, \\ \dots & \end{array} \}$$

- $G_{KJ} = \{\textcolor{blue}{HLL}, \text{HH}, \text{HLH}, \text{LL}\# \}$

\*#**HLLL**L#, \*#**HLL**HL#

## Computational locality

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\*#HLLLL#, \*#HLLHL#, \*#LLHHL#

## Computational locality

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►  $G_{KJ} = \{\text{HLL, HH, HLH, LL}\#\}$

\*#HLLLL#, \*#HLLHL#, \*#LLHHL#, \*#L**HLHL**#

## Computational locality

$$KJ = \{ \begin{array}{ll} \#HL\#, & \#LH\#, \\ \#LHL\#, & \#LLH\#, \\ \#LLHL\#, & \#LLLH\#, \\ \dots & \end{array} \}$$

►  $G_{KJ} = \{\text{HLL, HH, HLH, LL\#}\}$

\*#HLLLL#, \*#HLLHL#, \*#LLHHL#, \*#LHLHL#, \*#LLL**LL**\#, ...

# Computational locality

## Learning SL patterns

- ▶ Fix  $k$

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$$\text{substr}_k(w) = \{u \mid u \text{ is a } k\text{-substring of } w\}$$

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- ▶  $\text{substr}_3(\#LLHL\#) = \{\#LL, LLH, LHL, HL\#\}$

# Computational locality

## Learning SL patterns

- ▶ Fix  $k$

$$\text{substr}_k(w) = \{u \mid u \text{ is a } k\text{-substring of } w\}$$

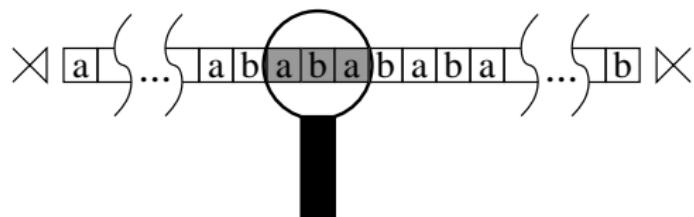
$$\text{substr}_k(D) = \bigcup_{w \in D} \text{substr}_k(w)$$

- ▶  $\text{substr}_3(\#LLHL\#) = \{\#LL, LLH, LHL, HL\#\}$

# Computational locality

## Learning SL patterns

- ▶  $\text{substr}_k$  scans each input string with a window of size  $k$



Rogers and Pullum (2011); Rogers et al. (2013)

# Computational locality

## Learning SL patterns

- ▶ Let  $S_k$  be the set of all possible  $k$ -substrings
- ▶ Learner:  $G_0 = S_k$   
$$G_n = S_k - \text{substr}_k(\{d_1, d_2, \dots, d_n\})$$

# Computational locality

## Learning SL patterns

- ▶ Let  $S_k$  be the set of all possible  $k$ -substrings

- ▶ Learner:  $G_0 = S_k$

$$G_n = S_k - \text{substr}_k(\{d_1, d_2, \dots, d_n\})$$

- ▶ KJ:  $k = 3$

$$G_0 = \{ \#LL, LLL, LLH, LHL, HLL, HLH, \dots, LH\# \}$$

# Computational locality

## Learning SL patterns

- ▶ Let  $S_k$  be the set of all possible  $k$ -substrings

- ▶ Learner:  $G_0 = S_k$

$$G_n = S_k - \text{substr}_k(\{d_1, d_2, \dots, d_n\})$$

- ▶ KJ:  $k = 3$

$$G_1 = \{ \#LL, LLL, LLH, LHL, HLL, HLH, \dots, LH\# \}$$

<u>Time</u>	<u>Datum</u>	<u>substr<sub>3</sub></u>
1	#LLHL#	{#LL, LLH, LHL, HL#}

# Computational locality

## Learning SL patterns

- ▶ Let  $S_k$  be the set of all possible  $k$ -substrings

- ▶ Learner:  $G_0 = S_k$

$$G_n = S_k - \text{substr}_k(\{d_1, d_2, \dots, d_n\})$$

- ▶ KJ:  $k = 3$

$$G_2 = \{ \#LL, LLL, LLH, LHL, HLL, HLH, \dots, LH\# \}$$

Time	Datum	<u>substr<sub>3</sub></u>
1	#LLHL#	{#LL, LLH, LHL, HL#}
2	#LLLH#	{#LL, LLL, LLH, LH#}

# Computational locality

## Learning SL patterns

- ▶ Let  $S_k$  be the set of all possible  $k$ -substrings

- ▶ Learner:  $G_0 = S_k$

$$G_n = S_k - \text{substr}_k(\{d_1, d_2, \dots, d_n\})$$

- ▶ KJ:  $k = 3$

$$G_3 = G_2 = \{ \#LL, LLL, LLH, LHL, HLL, HLH, \dots, LH\# \}$$

Time	Datum	$\text{substr}_3$
1	#LLHL#	{#LL, LLH, LHL, HL#}
2	#LLLH#	{#LL, LLL, LLH, LH#}
3	#LLLLHL#	{#LL, LLL, LLH, LHL, HL#}

# Computational locality

## Learning SL patterns

- ▶ Let  $S_k$  be the set of all possible  $k$ -substrings

- ▶ Learner:  $G_0 = S_k$

$$G_n = S_k - \text{substr}_k(\{d_1, d_2, \dots, d_n\})$$

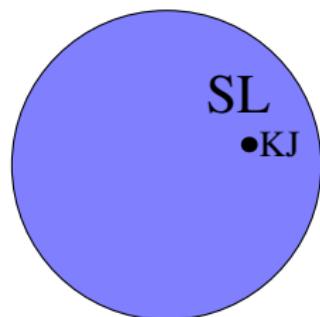
- ▶ *KJ*:  $k = 3$

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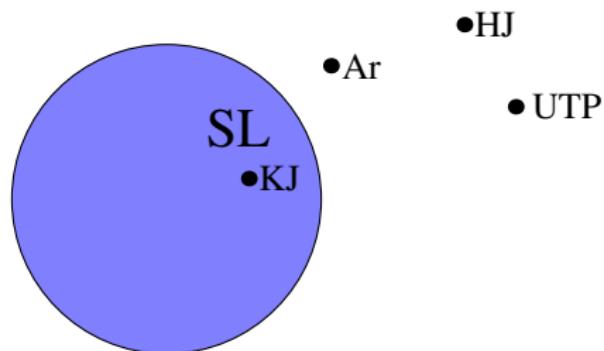
Time	Datum	<u>substr<sub>3</sub></u>
1	#LLHL#	{#LL, LLH, LHL, HL#}
2	#LLLH#	{#LL, LLL, LLH, LH#}
3	#LLLLHL#	{#LL, LLL, LLH, LHL, HL#}
...		

- ▶ There will always be some  $n$  for which  $G_n$  describes *KJ* (García et al., 1990; Heinz, 2010, 2011)

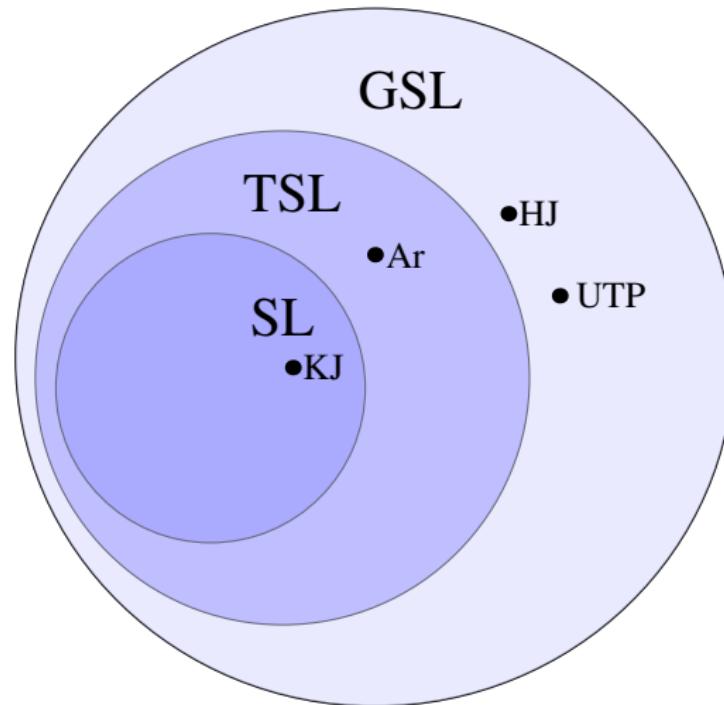
## Tone patterns and stringset classes



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# Non-local tone patterns

► Arigibi (Donohue, 1997): **At most one H**

a.	na:	'finish'	e.	umú	'dog'	j.	ola?olá	'red'
	L			LH			LLLH	
b.	tutu:	'long'	f.	nímo	'louse'	k.	tuni?á?ʌ	'all'
	LL			HL			LLHL	
c.	vovo?o	'bird'	g.	mudebé	'claw'	l.	idómai	'eye'
	LLL			LLH			LHLL	
d.	ɛlaila	'hot'	h.	ivío	'sun'	m.	nú?ʌtama	'bark'
	LLLL			LHL			HLLL	
			i.	ŋgí?ɛpu	'heart'			
				HLL				

# Non-local tone patterns

- Arigibi (Donohue, 1997): **At most one H**

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	LL			HL			LLHL	
c.	vovo?o	'bird'	g.	mudebé	'claw'	l.	idómai	'eye'
	LLL			LLH			LHLL	
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	LLLL			LHL			HLLL	
			i.	ŋgí?ɛpu	'heart'			
				HLL				

- Not SL
- \*H...H; {HH, HLH, HLLH, HLLLH, ...}

# Non-local tone patterns

- ▶ Arigibi: **At most one H**
- ▶ Hirosaki J. (Haraguchi, 1977):  
**Exactly one H or F; F word-final**

a. ‘chicken’	niwatorí	LLLH
b. ‘lightening’	kaminarî	LLLF
c. ‘fruit’	kudamóno	LLHL
d. ‘trunk’	toránku	LHLL
e. ‘bat’	kóomori	HLLL

# Non-local tone patterns

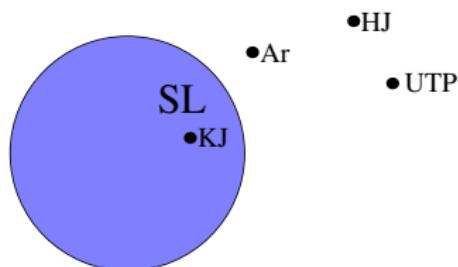
- ▶ Arigibi: **At most one H**
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**Exactly one H or F; F word-final**
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  - b. ‘lightening’    kaminarî      LLLF
  - c. ‘fruit’           kudamóno      LLHL
  - d. ‘trunk’           toránku      LHLL
  - e. ‘bat’              kóomori      HLLL
- ▶ \*H...H, \*H...F, \*L<sup>n</sup>, \*FL, etc.

## Non-local tone patterns

- ▶ Arigibi: **At most one H**
  - ▶ Hirosaki J.: **Exactly one H or F; F word-final**
  - ▶ Unbounded Tone Plateauing (Hyman, 2011; Jardine, to appear):  
**At most one plateau of H**
    - a. ‘chopper’      mutéma      LHL
    - b. ‘log’            kisikí        LLH
    - c. ‘log chopper’    mutémá+bísíkí    LHHHHH
    - d. // /             \*mutéma+bisikí    \*LHLLLH
- (Luganda; Hyman, 2011; Hyman and Katamba, 2010)

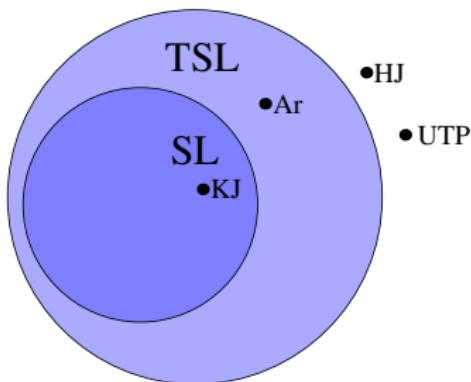
# Non-local tone patterns

- ▶ Arigibi: **At most one H** (Ar)
- ▶ Hirosaki J.: **Exactly one H or F; F word-final** (HJ)
- ▶ Unbounded Tone Plateauing: **At most one plateau of H** (UTP)



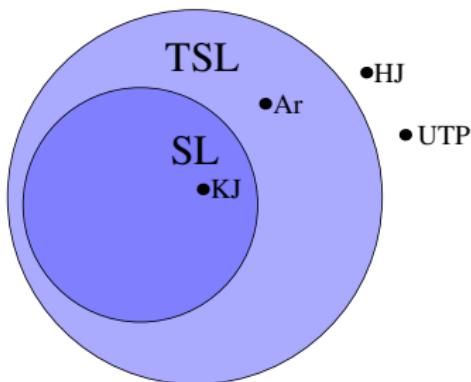
## Non-local tone patterns

- ▶ **Tier-based SL** (Heinz et al., 2011): Banned substrings evaluated over one subset or **tier** of alphabet
- ▶ Arigibi:  $G = \{HH\}$  when L is ignored;  
\*HLH, \*LLHLLLH, ...



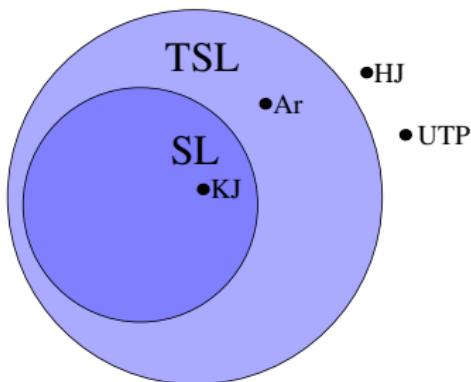
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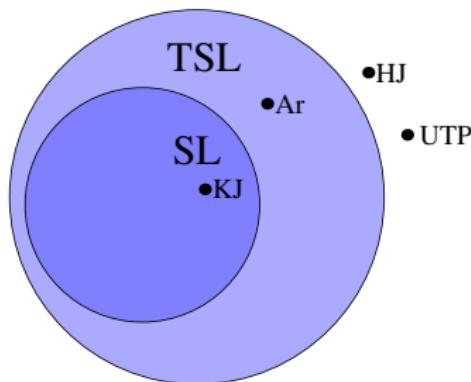
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 $*\textcolor{red}{H}\textcolor{black}{L}\textcolor{red}{H}$ ,  $*\textcolor{red}{L}\textcolor{black}{L}\textcolor{red}{H}\textcolor{black}{L}\textcolor{black}{L}\textcolor{red}{H}$ , ...



## Non-local tone patterns

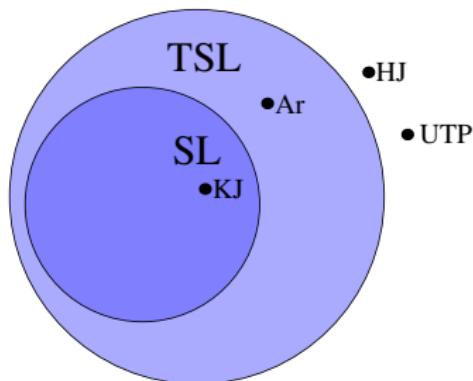
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- ▶ Arigibi:  $G = \{HH\}$  when L is ignored;  
 $*\textcolor{red}{H}\textcolor{black}{L}\textcolor{red}{H}$ ,  $*\textcolor{red}{L}\textcolor{black}{L}\textcolor{red}{H}\textcolor{black}{L}\textcolor{black}{L}\textcolor{red}{H}$ , ...



- ▶ Tier and grammar are learnable (Jardine and Heinz, 2016)

## Non-local tone patterns

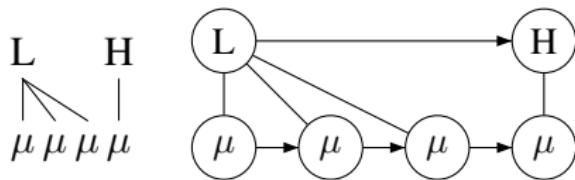
- ▶ For *HJ* (exactly one H; F word-final), can't ignore Ls: \*FLL...
- ▶ For *UTP* (at most one H plateau), can't posit \*HH



# Computational locality for autosegmental representations

- ▶ Autosegmental representations are **graphs** (Goldsmith, 1976; Coleman and Local, 1991)

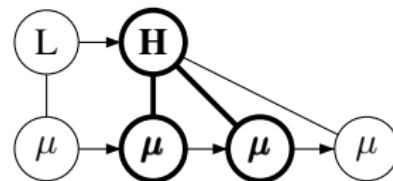
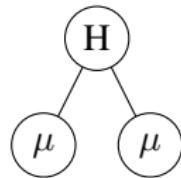
ola?olá LLLH ‘red’  
(Arigibi)



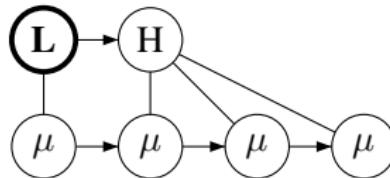
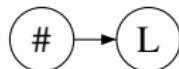
- ▶ We can instead consider banned subgraph grammars

# Computational locality for autosegmental representations

- Let a **subgraph** be some finite, connected piece of a graph



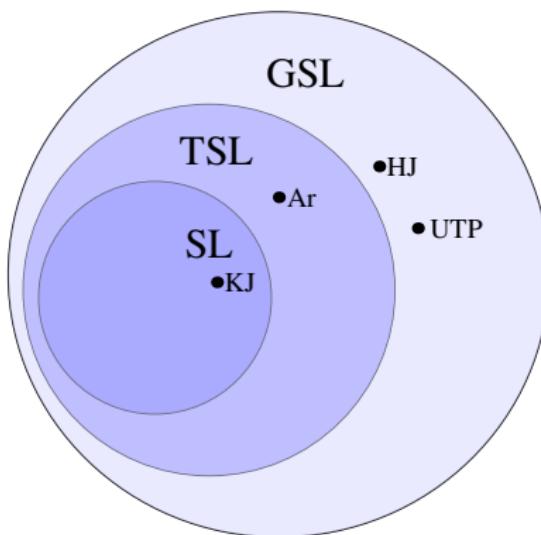
- Subgraphs may refer to boundaries on each tier (not depicted in full graphs)



- $k$  is the number of nodes

# Computational locality for autosegmental representations

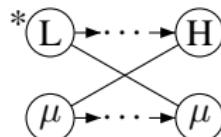
- ▶ **Graph SL:** Describable by banned subgraph grammars



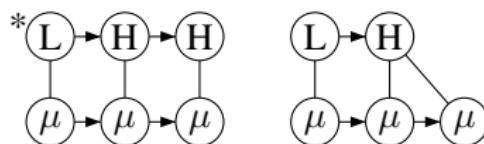
- ▶ These grammars are learnable in a similar way to SL grammars

## Some assumptions

- ▶ Association preserves precedence relations (**the No-Crossing Constraint (NCC)**)



- ▶ Adjacent nodes on tonal tier cannot be identical (**the Obligatory Contour Principle (OCP)**)

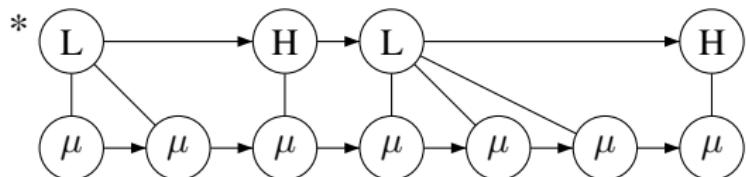
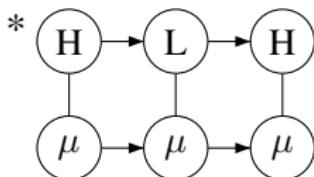


- ▶ Such representations can be generated from strings (Jardine and Heinz, 2015)

# Banned subgraph analyses

**Arigibi: At most one H**

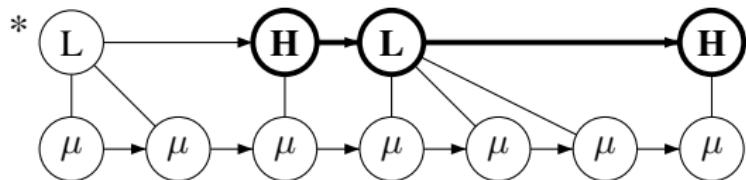
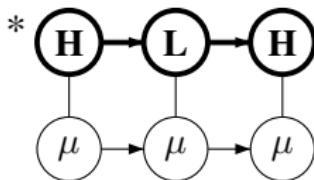
- ▶ \*HLH, \*HLLH, \*LLHLLLH, ...



# Banned subgraph analyses

**Arigibi: At most one H**

- \*HLH, \*HLLH, \*LLHLLLH, ...

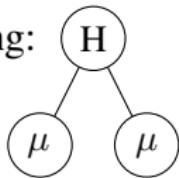


- First banned subgraph:

# Banned subgraph analyses

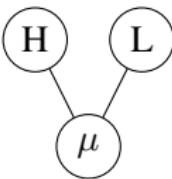
## Arigibi: At most one H

► Spreading:

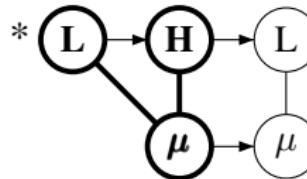
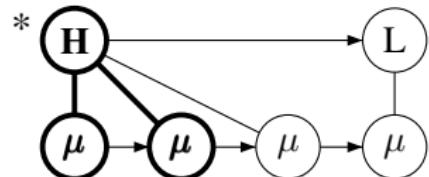


\*HHHL

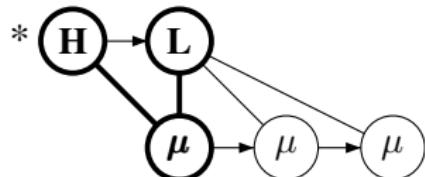
► Contours:



\*RL



\*FLLL



# Banned subgraph analyses

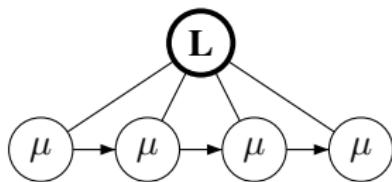
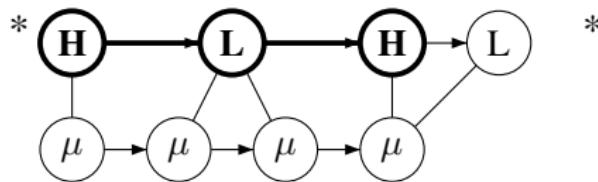
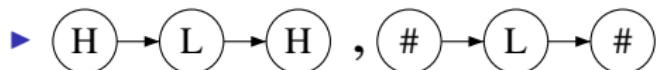
**Arigibi: At most one H**

- ▶  $G_{Ar} = \left\{ \begin{array}{c} \text{H} \rightarrow \text{L} \rightarrow \text{H} \\ , \end{array}, \begin{array}{c} \text{H} \\ | \\ \mu \quad \mu \end{array}, \begin{array}{c} \text{H} \\ | \\ \mu \end{array}, \begin{array}{c} \text{L} \\ | \\ \mu \end{array} \right\}$

# Banned subgraph analyses

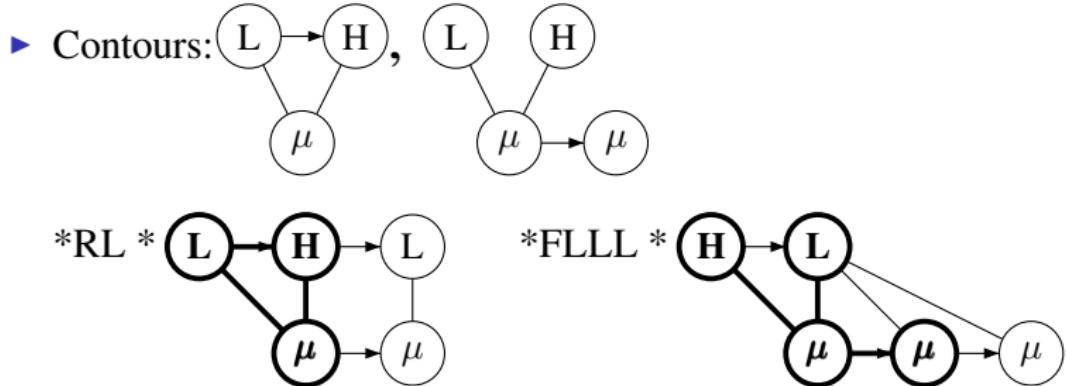
**Hirosaki Japanese: Exactly one H or F; F is word-final**

- ▶ \*HLLF, \*LLLL



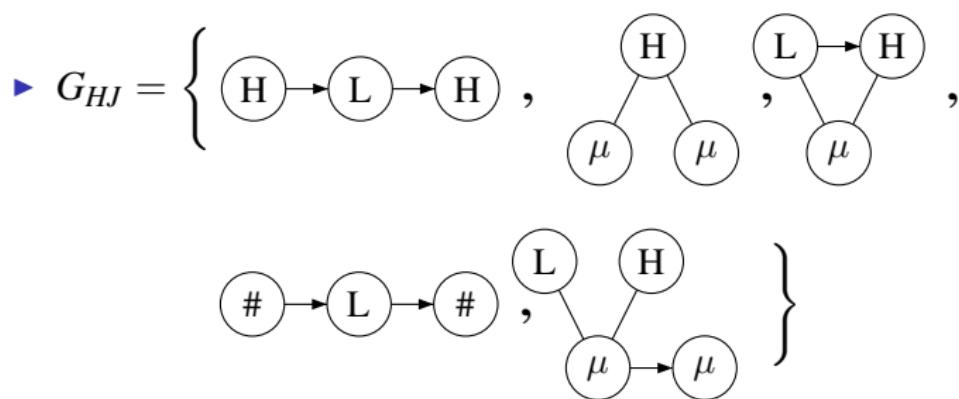
# Banned subgraph analyses

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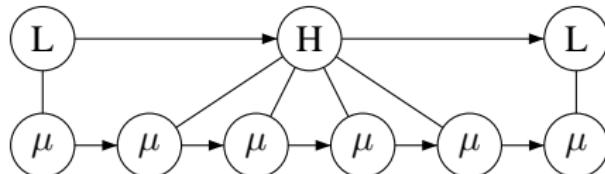
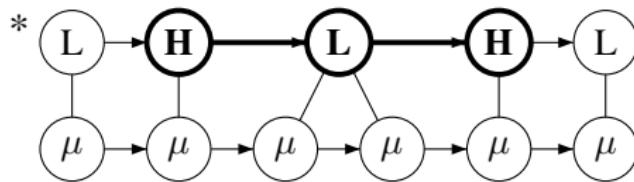


# Banned subgraph analyses

**UTP: At most one plateau of H**

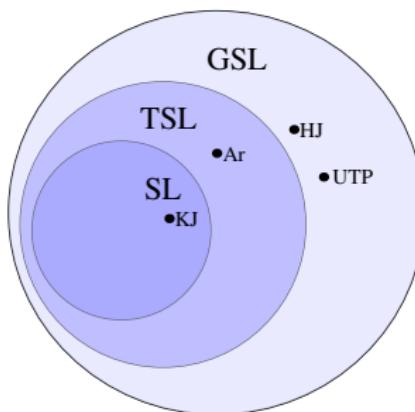
$$\triangleright G_{UTP} = \left\{ \begin{array}{c} \text{H} \rightarrow \text{L} \rightarrow \text{H}, \\ \text{H} \text{---} \text{L} \text{---} \mu \end{array} \right\}$$

$\triangleright *LHLLHL, LHHHHL$



# Banned subgraph analyses

- ▶ Arigibi: **At most one H** (Ar)
- ▶ Hirosaki J.: **Exactly one H or F; F word-final** (HJ)
- ▶ Unbounded Tone Plateauing: **At most one plateau of H** (UTP)



# Learning GSL

- ▶ Fix  $k$

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$$\text{subg}_k(g) = \{s \mid s \text{ is a } k\text{-subgraph of } g\}$$

# Learning GSL

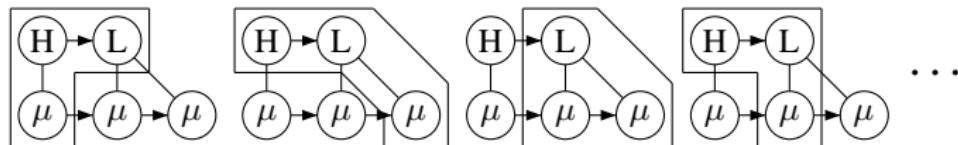
- ▶ Fix  $k$

$$\text{subg}_k(g) = \{s \mid s \text{ is a } k\text{-subgraph of } g\}$$

$$\text{subg}_k(D) = \bigcup_{g \in D} \text{subg}_k(g)$$

# Learning banned GSL

- ▶ Cognitive interpretation of  $\text{subg}_k$  is the same as  $\text{substr}_k$ : scan input structures, remembering substructures of size  $k$



## Learning GSL

- ▶ Let  $S_k$  be the set of all possible *k*-subgraphs
- ▶ Learner:  $G_0 = S_k$   
 $G_n = S_k - \text{subg}_k(\{d_1, d_2, \dots, d_n\})$

# Learning GSL

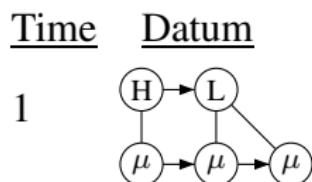
- ▶ Let  $S_k$  be the set of all possible  $k$ -subgraphs
- ▶ Learner:  $G_0 = S_k$   
$$G_n = S_k - \text{subg}_k(\{d_1, d_2, \dots, d_n\})$$
- ▶ Ar:  $k = 3$

$$G_0 = \left\{ \begin{array}{c} \text{H} \\ \diagdown \quad \diagup \\ \mu \quad \mu \end{array}, \begin{array}{c} \text{L} \\ \diagdown \quad \diagup \\ \mu \quad \mu \end{array}, \dots, \begin{array}{c} \text{L} \rightarrow \text{H} \rightarrow \text{L} \\ \text{H} \rightarrow \text{L} \rightarrow \text{H} \end{array} \right\}$$

# Learning GSL

- ▶ Let  $S_k$  be the set of all possible  $k$ -subgraphs
- ▶ Learner:  $G_0 = S_k$   
 $G_n = S_k - \text{subg}_k(\{d_1, d_2, \dots, d_n\})$
- ▶ Ar:  $k = 3$

$$G_1 = \left\{ \begin{array}{c} \text{H} \\ \diagdown \quad \diagup \\ \mu \quad \mu \end{array}, \begin{array}{c} \text{L} \\ \diagdown \quad \diagup \\ \mu \quad \mu \end{array}, \dots, \begin{array}{c} \text{L} \rightarrow \text{H} \rightarrow \text{L} \\ \text{H} \rightarrow \text{L} \rightarrow \text{H} \end{array} \right\}$$

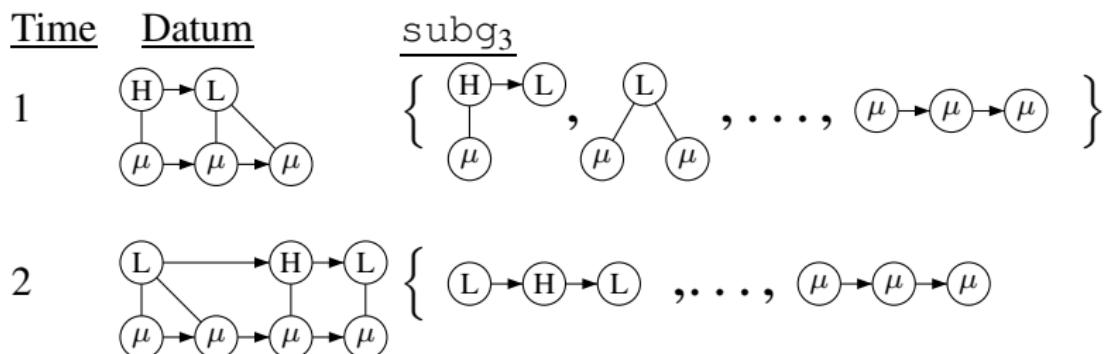


$$\text{subg}_3 \left\{ \begin{array}{c} \text{H} \rightarrow \text{L} \\ \diagdown \quad \diagup \\ \mu \quad \mu \end{array}, \begin{array}{c} \text{L} \\ \diagdown \quad \diagup \\ \mu \quad \mu \end{array}, \dots, \begin{array}{c} \mu \rightarrow \mu \rightarrow \mu \end{array} \right\}$$

# Learning GSL

- Let  $S_k$  be the set of all possible  $k$ -subgraphs
- Learner:  $G_0 = S_k$   
 $G_n = S_k - \text{subg}_k(\{d_1, d_2, \dots, d_n\})$
- Ar:  $k = 3$

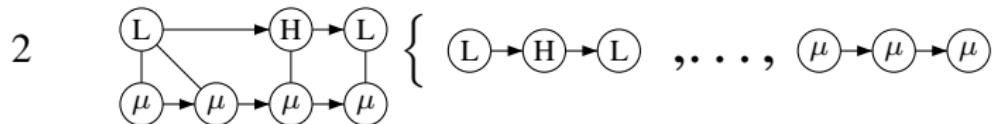
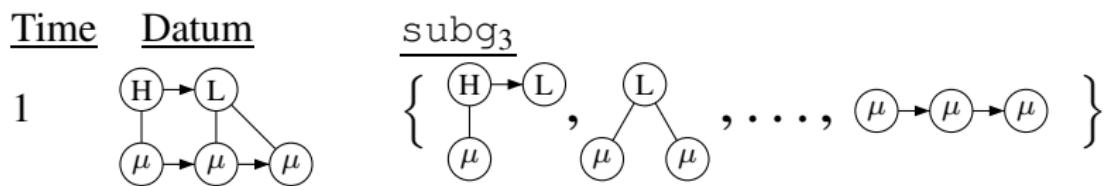
$$G_2 = \left\{ \begin{array}{c} \text{H} \\ \diagdown \quad \diagup \\ \mu \quad \mu \end{array}, \begin{array}{c} \text{L} \\ \diagdown \quad \diagup \\ \mu \quad \mu \end{array}, \dots, \begin{array}{c} \text{L} \rightarrow \text{H} \rightarrow \text{L} \\ \text{H} \rightarrow \text{L} \rightarrow \text{H} \end{array} \right\}$$



# Learning GSL

- Let  $S_k$  be the set of all possible  $k$ -subgraphs
- Learner:  $G_0 = S_k$   
 $G_n = S_k - \text{subg}_k(\{d_1, d_2, \dots, d_n\})$
- At:  $k = 3$

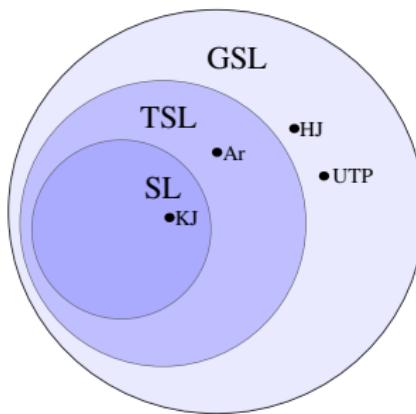
$$G_2 = \left\{ \begin{array}{c} \text{H} \\ \mu \quad \mu \end{array}, \begin{array}{c} \text{L} \\ \mu \quad \mu \end{array}, \dots, \begin{array}{c} \text{L} \rightarrow \text{H} \rightarrow \text{L} \\ \text{H} \rightarrow \text{L} \rightarrow \text{H} \end{array} \right\}$$



...

# Learning GSL

- ▶ There will always be some  $n$  for which  $G_n$  describes  $Ar$
- ▶ Searching for  $k$  connected subgraphs is tractable (Ferreira, 2013)
- ▶ Can learn from string input (Jardine and Heinz, 2015)



# Conclusions

- ▶ Tone includes many long-distance patterns, including some that are outside of the range of established string-based learners
- ▶ A graph-based learner can learn these patterns
- ▶ This is thanks to a computational notion of **locality** extended to autosegmental representations
- ▶ Future work: how can this result be extended to representations in other domains (segmental, metrical)?

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