Logical Characterizations of Phonological Patterns

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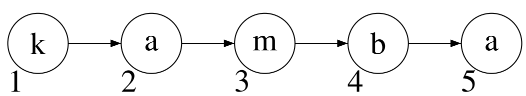
Models

String model defined over immediate **successor** ($\triangleleft$) and some finite alphabet $\{a, b, ..., z\}$ $\triangleleft$ (successor)

$$\langle W, \triangleleft, P_a, P_b, ..., P_z \rangle$$

Ex. [kamba]

$$\langle \quad \{1,2,3,4,5\}_W, \\ \{(1,2),(2,3),(3,4),(4,5)\}_{\triangleleft}, \\ \{2,5\}_a, \{1\}_k, \{3\}_m, \{4\}_p \quad \rangle$$

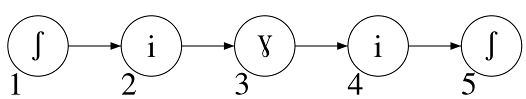


String model defined over **precedence** ($<$) and some finite alphabet $<$ (precedence)

$$\langle W, <, P_a, P_b, ..., P_z \rangle$$

Ex. [ʃiɣiʃ]

$$\langle \quad \{1,2,3,4,5\}_W, \\ \{(1,2),(1,3),(1,4),(1,5),(2,3),(2,4),(2,5),(3,4),(3,5),(4,5)\}_<, \\ \{1,5\}_{\text{ʃ}}, \{2,4\}_i, \{4\}_{\text{ɣ}}, \quad \rangle$$



Logical statements

By $\mathcal{X} = \{x, y, ..., z, x_1, x_2, ...\}$ we refer to some set of variables. We define the **syntax** of $FO^{\triangleleft}$, or first order logic for $\triangleleft$ string models, recursively as in Def. 1. This explicitly defines the range of constraints we can write in $FO^{\triangleleft}$.

$\mathcal{X}$ (set of variables)
syntax of $FO^{\triangleleft}$

Definition 1 (Syntax of $FO^{\triangleleft}$) *A statement φ is in $FO^{\triangleleft}$ iff one of the following holds:*

- $\varphi = a(x)$ *for some* a *in the alphabet*
- $\varphi = x \triangleleft y$
- $\varphi = \neg(\psi)$ *for some* $\psi \in FO^{\triangleleft}$
- $\varphi = (\psi_1 \wedge \psi_2)$ *for some* $\psi_1, \psi_2 \in FO^{\triangleleft}$
- $\varphi = \psi_1 \vee \psi_2$ *for some* $\psi_1, \psi_2 \in FO^{\triangleleft}$
- $\varphi = \psi_1 \rightarrow \psi_2$ *for some* $\psi_1, \psi_2 \in FO^{\triangleleft}$
- $\varphi = (\forall x_1, x_2, ..., x_n)[\psi]$ *for some* $\psi \in FO^{\triangleleft}$
- $\varphi = (\exists x_1, x_2, ..., x_n)[\psi]$ *for some* $\psi \in FO^{\triangleleft}$

No other statement is in $FO^{\triangleleft}$.

We omit parentheses when their interpretation is clear.
For some $\triangleleft$ string model $\mathcal{W}$ whose set of positions is W, let $S : \mathcal{X} \rightarrow W$ be a function assigning variables to positions in W. The **semantics** of $FO^{\triangleleft}$ is defined by how $\mathcal{W}$ can **satisfy** a statement $\varphi \in FO^{\triangleleft}$, written $\mathcal{W} \models \varphi$.

S (assignment function)
semantics of $FO^{\triangleleft}$
$\mathcal{W} \models \varphi$ (satisfaction)

Definition 2 (Semantics of $FO^{\triangleleft}$) *A word $\mathcal{W}$ satisfies φ, written $\mathcal{W} \models \varphi$, when φ is a statement prefixed with a quantifer and one of the following holds:*

- $\varphi = (\forall x_1, x_2, ..., x_n)[\psi]$ *and* $\mathcal{W}, S \models \psi$ *for* all *assignments* S *mapping* $x_1, x_2, ..., x_n$ *to positions in* W
- $\varphi = (\exists x_1, x_2, ..., x_n)[\psi]$ *and* $\mathcal{W}, S \models \psi$ *for* some *assignment* S *mapping* $x_1, x_2, ..., x_n$ *to positions in* W

where $\mathcal{W}, S \models \varphi$ *($\mathcal{W}$ satisfies φ given a particular assignment S) is defined as follows:*

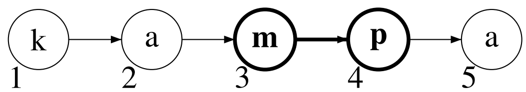
- $\mathcal{W}, S \models a(x)$ *iff* $S(x) \in P_a$ *in* $\mathcal{W}$
- $\mathcal{W}, S \models x \triangleleft y$ *iff* $(S(x), S(y)) \in \triangleleft$ *in* $\mathcal{W}$
- $\mathcal{W}, S \models \neg\varphi$ *iff* $\mathcal{W}, S$ *does not satisfy* φ
- $\mathcal{W}, S \models \varphi \vee \psi$ *iff* $\mathcal{W}, S \models \varphi$ *or* $\mathcal{W}, S \models \psi$
- $\mathcal{W}, S \models \varphi \wedge \psi$ *iff* $\mathcal{W}, S \models \neg(\neg\varphi \vee \neg\psi)$
- $\mathcal{W}, S \models \varphi \rightarrow \psi$ *iff* $\mathcal{W}, S \models \neg\varphi \vee \psi$

For a more thorough introduction, see Enderton (1972).

Examples

- *NC̥

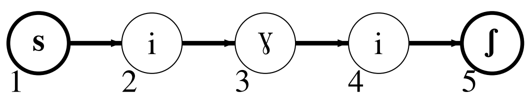
$$nasal(x) \equiv m(x) \vee n(x) \vee ... \vee \eta(x)$$
$$voiceless(x) \equiv p(x) \vee t(x) \vee ... \vee k(x)$$
$$\varphi_{*\text{NC̥}} = \forall(x,y)[(x \triangleleft y \wedge nasal(x)) \rightarrow \neg voiceless(y)]$$



$\varphi_{*\text{NC̥}}$ is not true for an S where $S(x) = 3$ and $S(y) = 4$

- *s...ʃ using $FO^{<}$

$$+AntSib(x) \equiv s(x) \vee z(x) \vee ts(x)$$
$$-AntSib(x) \equiv \text{ʃ}(x) \vee \text{ʒ}(x) \vee \text{tʃ}(x)$$
$$\varphi_{*\text{s...ʃ}} \equiv (\forall x, y)[(x < y \wedge +AntSib(x)) \rightarrow \neg - AntSib(y)]$$



- Post-nasal voicing with graph transductions (Engelfriet and Hoogeboom, 2001)

$$mp(x) \equiv p(x) \wedge (\exists y)[m(y) \wedge y \triangleleft x]$$

Node formulae

$\varphi^0_a(x) \equiv a(x)$

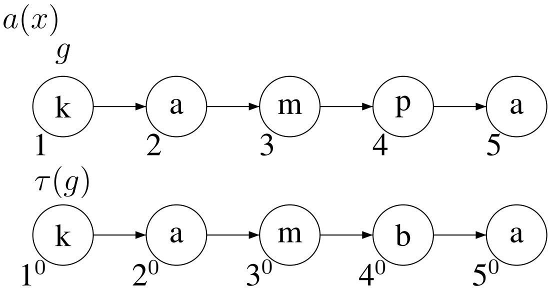
$\varphi^0_m(x) \equiv m(x)$

$\varphi^0_b(x) \equiv b(x) \vee mp(x)$

$\varphi^0_p(x) \equiv p(x) \wedge \neg mp(x)$

Edge formula

$\varphi^0_a(x) \equiv a(x)$



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