

# Computational Locality and Autosegmental Processes

Jane Chandlee and Adam Jardine

Haverford College

Rutgers University

CLS 54

26 April 2018

# Introduction

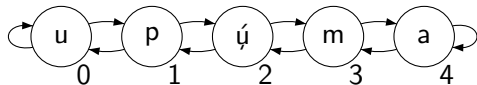
- ▶ Autosegmental representations (ARs) (Goldsmith, 1976; Clements, 1976) have been claimed to capture non-local phenomena in a local way (McCarthy, 1982; Odden, 1994).
- ▶ We apply a computational notion of locality to a selection of tone processes to get a more nuanced understanding of this ability of ARs.
- ▶ Three-way distinction:
  - ▶ Local even without ARs.
  - ▶ Local only with ARs.
  - ▶ Not local even with ARs.

# Computational notion of locality

- ▶ Based on the Input Strictly Local (ISL) functions, which were originally defined in terms of formal language theory and automata theory (Chandlee, 2014).
- ▶ We'll be using the logical characterization of ISL proposed by Chandlee and Lindell (to appear).
  - ▶ ISL function = Quantifier-free First Order Graph Interpretation
- ▶ Why use logic?
  - ▶ We can directly extend a restrictive, explicit notion of locality from strings to phonological representations

## FO Graph interpretations

- (1) Rimi (Schadeberg, 1979; Meyers, 1997)  
/u-púm-a/  $\mapsto$  [u-púm-á] 'to go away'

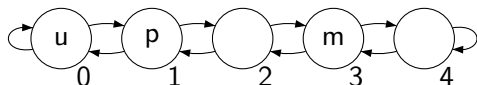


UR string model defined with:

- ▶ 5 positions, labeled with segments
- ▶ successor function:  $s(0) = 1, s(1) = 2, \dots, s(4)=4$
- ▶ predecessor function  $p(4) = 3, p(3) = 2, \dots, p(0)=0$

## FO Graph interpretations

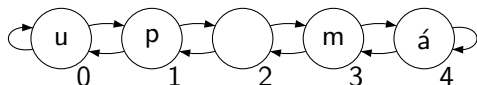
- ▶ The SR string graph is defined in terms of the UR graph using FO logic formulas (Engelfriet and Hoogeboom, 2001).



- ▶  $H(x)$  is True iff position  $x$  bears a high tone.
- ▶  $V(x)$  is True iff position  $x$  is a vowel.
- ▶  $\varphi_V \stackrel{\text{def}}{=} V(x) \wedge H(p(p(x)))$ 
  - ▶ An output position bears a high tone iff it's a vowel and the previous vowel bears a high tone in the input graph.

## FO Graph interpretations

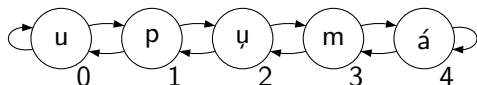
- ▶ The SR string graph is defined in terms of the UR graph using FO logic formulas (Engelfriet and Hoogeboom, 2001).



- ▶  $H(x)$  is True iff position  $x$  bears a high tone.
- ▶  $V(x)$  is True iff position  $x$  is a vowel.
- ▶  $\varphi_{\dot{V}} \stackrel{\text{def}}{=} V(x) \wedge H(p(p(x)))$ 
  - ▶ An output position bears a high tone iff it's a vowel and the previous vowel bears a high tone in the input graph.

## FO Graph interpretations

- ▶ The SR string graph is defined in terms of the UR graph using FO logic formulas (Engelfriet and Hoogeboom, 2001).



- ▶  $H(x)$  is True iff position  $x$  bears a high tone.
- ▶  $V(x)$  is True iff position  $x$  is a vowel.
- ▶  $\varphi_{\dot{V}} \stackrel{\text{def}}{=} V(x) \wedge H(p(p(x)))$ 
  - ▶ An output position bears a high tone iff it's a vowel and the previous vowel bears a high tone in the input graph.

## ISL = Quantifier-free FO Logic

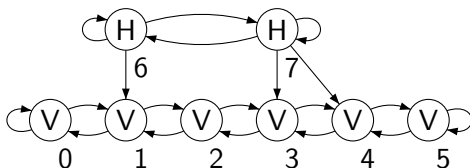
$$\varphi_V \stackrel{\text{def}}{=} V(x) \wedge H(p(p(x)))$$

- ▶ These formulas do not use the full power of FO: they don't use quantifiers.
- ▶ The processes that can be described in this way are those for which the trigger and the target form a **contiguous substring of bounded length** in the input string.
- ▶ The boundedness means we can use the successor or predecessor function repeatedly to determine whether both the target and trigger are present.
  - ▶ No quantifier is needed.  
(e.g, no "...  $\wedge (\exists z)[\dots]$ ")

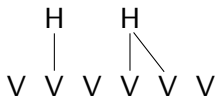
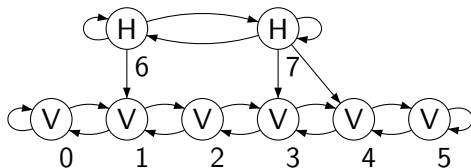


# Autosegmental Representation (AR) Graphs

- ▶ Goal: extend this same notion of locality (QF FO describable) from string graphs to AR graphs (Jardine, 2016).
- ▶ Example:  $V\acute{V}V\acute{V}V$



# Autosegmental Representation (AR) Graphs



## Method

- ▶ We designate as ISL those patterns that can be describe with QF FO using string graphs.
- ▶ We designate as AISL those patterns that can be described with QF FO using AR graphs.
- ▶ We illustrate that ISL patterns are also AISL but not vice versa
- ▶ We will also identify cases that are neither ISL nor AISL.

## AISL Analyses

<b>Process</b>	<b>ISL?</b>	<b>AISL?</b>
Bounded spread (Bemba)	✓	
Bounded shift (Rimi)	✓	
Unbounded shift (Zigula)	✗	
Unbounded OCP (Arusa)	✗	
Unbounded spread (Ndebele)	✗	
Meussen's rule (Shona)	✗	

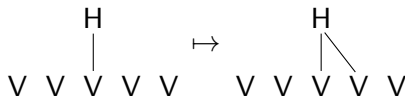
# AISL Analyses

## Bounded spread

- ▶ Bemba (Bickmore and Kula, 2013)

/bá-la-kak-a/      ⇨    [bá-lá-kak-a] 'they tie up'

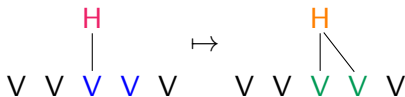
/tu-la-bá-kak-a/    ⇨    [tu-la-bá-kák-a] 'we tie them up'



# AISL Analyses

## Bounded spread

$$a_O(x, y) \stackrel{\text{def}}{=} a_I(x, y) \vee a_I(p(x), y)$$



# AISL Analyses

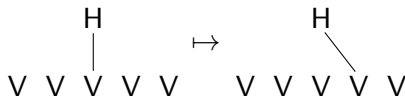
## Bounded shift

- ▶ Rimi (Schadeberg, 1979; Meyers, 1997)

/u-pùm-a/    ⇨    [u-pùm-á]    'to go away'

/rá-mu-ntu/    ⇨    [ra-mú-ntu]    'of a person'

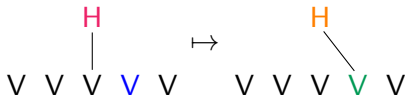
/mu-tém-j/    ⇨    [mu-tem-í]    'chief'



# AISL Analyses

## Bounded shift

$$a_O(x, y) \stackrel{\text{def}}{=} a_I(p(x), y)$$





## AISL Analyses

<b>Process</b>	<b>ISL?</b>	<b>AISL?</b>
Bounded spread (Bemba)	✓	✓
Bounded shift (Rimi)	✓	✓
Unbounded shift (Zigula)	✗	
Unbounded OCP (Arusa)	✗	
Unbounded spread (Ndebele)	✗	
Meussen's rule (Shona)	✗	

## Unbounded shift

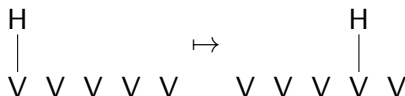
- ▶ Zigula (Kenstowicz and Kisseberth, 1990)

ku-gulus-a            'to chase'

ku-lombéz-a         'to ask'

ku-lombež-éz-a      'to ask for'

ku-lombež-ež-án-a   'to ask for each other'

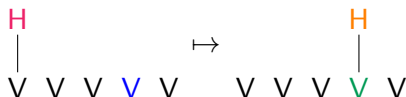


# AISL Analyses

## Unbounded shift

$$\text{lastH}(y) \stackrel{\text{def}}{=} H(y) \wedge s(y) = y$$
$$\text{penultV}(x) \stackrel{\text{def}}{=} V(x) \wedge (s(s(x)) = s(s(s(x))))$$

$$a_0(x, y) \stackrel{\text{def}}{=} \text{penultV}(x) \wedge \text{LastH}(y)$$



# AISL Analyses

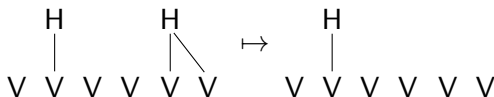
## Unbounded OCP

- ▶ Arusa (Odden, 1994)

sídáy 'good'

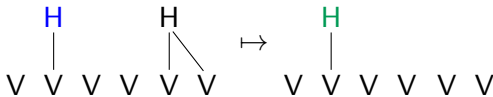
enkér siday 'good ewe'

olórika siday 'good chair'



## Unbounded OCP

$$H_O(x) \stackrel{\text{def}}{=} H(x) \wedge \neg H(p(x))$$



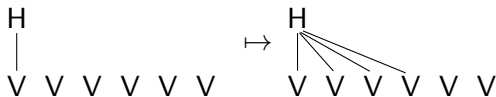
## AISL Analyses

<b>Process</b>	<b>ISL?</b>	<b>AISL?</b>
Bounded spread (Bemba)	✓	✓
Bounded shift (Rimi)	✓	✓
Unbounded shift (Zigula)	✗	✓
Unbounded OCP (Arusa)	✗	✓
Unbounded spread (Ndebele)	✗	
Meussen's rule (Shona)	✗	

# Unbounded spread

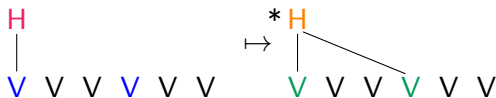
► Ndebele (Sibanda, 2004)

/ú-ku-hlek-a/	↦	[ú-kú-hlek-a]	'to laugh'
/ú-ku-hlek-is-a/	↦	[ú-kú-hlék-is-a]	'to amuse'
/ú-ku-hlek-is-an-a/	↦	[ú-kú-hlék-ís-an-a]	'to amuse e. o.'



## Unbounded spread

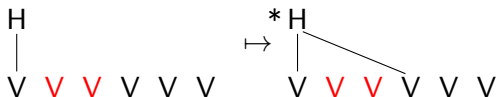
$$a_0(x, y) \stackrel{\text{def}}{=} a_1(x, y) \vee (\text{antepenult}V(x) \wedge H_1(y))$$





# Unbounded spread

- ▶ No QF statement can identify all **intermediate** vowels



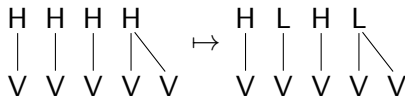
# Meussen's Rule

- ▶ Shona Odden (1986); Meyers (1987, 1997)

/né-hóvé/      ⇨    [né-hòvè]

/né-é-hóvé/    ⇨    [né-è-hóvé]

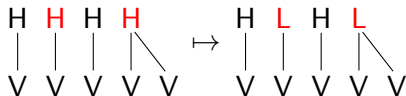
/né-é-é-hóvé/ ⇨    [né-è-é-hòvè]



## Meussen's Rule

- ▶ Need to pick out the set of even H's—this is well-known to be not definable even with (first-order) quantification (Thomas, 1982)

$$L_O \stackrel{\text{def}}{=} H_I(x) \wedge \text{even}(x)$$



## Summary

<b>Process</b>		<b>ISL?</b>	<b>AISL?</b>
Bounded spread	(Bemba)	✓	✓
Bounded shift	(Rimi)	✓	✓
Unbounded shift	(Zigula)	✗	✓
Unbounded OCP	(Arusa)	✗	✓
Unbounded spread	(Ndebele)	✗	✗
Meussen's rule	(Shona)	✗	✗

## Discussion

- ▶ Tone patterns include both ISL and non-ISL patterns

### **Unbounded shift:**

$$V \acute{V} V V V V \mapsto V V V V \acute{V} V$$

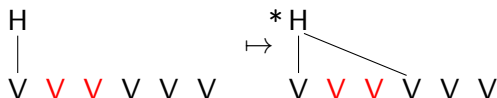
- ▶ With AISL, we can capture some non-ISL patterns

$$\begin{array}{cccccc} H & & & & & & H \\ | & & & & & & | \\ V & V & V & V & V & & V & V & V & V & V \end{array} \mapsto$$

- ▶ Thus, ARs make *some* non-local patterns local

# Discussion

## Unbounded spread:



- ▶ One option for non-ISL/AISL processes is to **further enrich** the representations
- ▶ We might consider AR models with  $<$  instead of  $p$  (generalization of Strictly Piecewise (Heinz, 2010; Rogers et al., 2010))

## Discussion

- ▶ Another option is to consider **output-based** locality
  - (2) Johore Malay (Onn, 1980)  
/pəŋawasan/  $\mapsto$  [pəŋãwãsan] ‘supervision’
- ▶ **Output SL** functions have been characterized for strings in terms of formal language and automata theory (Chandlee et al., 2015)
- ▶ A logical characterization of OSL remains for future work.

## Why do logical characterizations matter?

- ▶ Enable a rigorous, restrictive, and learnable (Chandlee and Heinz, 2018) definition of what it means to be “local” and “non-local”.
- ▶ Directly extend these notions from strings to ARs.
- ▶ Logics are tightly connected to the complexity of functions (Filiot and Reynier, 2016).
- ▶ Computational complexity classes have been shown to capture the typology of spreading (Heinz and Lai, 2013).



## Conclusion

- ▶ We *directly compared* different representations to better understand how ARs can render non-local processes local.
- ▶ Given an **input-based** notion of locality, ARs capture some, but not all, patterns that are non-local over strings.
- ▶ In future, an **output-based** notion of locality may accommodate additional processes that are not AISL.

## References I

- Bickmore, L. S. and Kula, N. C. (2013). Ternary spreading and the OCP in Copperbelt Bemba. *Studies in African Linguistics*, 42.
- Chandlee, J. (2014). *Strictly Local Phonological Processes*. PhD thesis, University of Delaware.
- Chandlee, J. and Heinz, J. (2018). Strictly locality and phonological maps. *LI*, 49:23–60.
- Chandlee, J., Jardine, A., and Heinz, J. (2015). Learning repairs for marked structures. In *Proceedings of the 2015 Annual Meeting on Phonology*. LSA.
- Chandlee, J. and Lindell, S. (to appear). A logical characterization of input strictly local functions. In Heinz, J., editor, *Doing Computational Phonology*. Oxford University Press.
- Clements, G. N. (1976). *Vowel Harmony in Nonlinear Generative Phonology: An Autosegmental Model*. Bloomington: Indiana University Linguistics Club Publications.

## References II

- Engelfriet, J. and Hoogeboom, H. J. (2001). MSO definable string transductions and two-way finite-state transducers. *ACM Trans. Comput. Logic*, 2(2):216–254.
- Filiot, E. and Reynier, P. (2016). Transducers, logic, and algebra for functions of finite words. *ACM SIGLOG News*, 3(3):4–19.
- Goldsmith, J. (1976). *Autosegmental Phonology*. PhD thesis, Massachusetts Institute of Technology.
- Heinz, J. (2010). Learning long-distance phonotactics. *LI*, 41:623–661.
- Heinz, J. and Lai, R. (2013). Vowel harmony and subsequentiality. In Kornai, A. and Kuhlmann, M., editors, *Proceedings of the 13th Meeting on Mathematics of Language*, Sofia, Bulgaria.
- Jardine, A. (2016). *Locality and non-linear representations in tonal phonology*. PhD thesis, University of Delaware.
- Kenstowicz, M. and Kisseberth, C. (1990). Chizigula tonology: the word and beyond. In Inkelas, S. and Zec, D., editors, *The Phonology–Syntax Connection*, pages 163–194. Chicago: the University of Chicago Press.
- McCarthy, J. (1982). Nonlinear phonology: An overview. *GLOW Newsletter*, 50.

## References III

- Meyers, S. (1987). *Tone and the structure of words in Shona*. PhD thesis, University of Massachusetts, Amherst.
- Meyers, S. (1997). OCP effects in Optimality Theory. *NLLT*, 15(4):847–892.
- Odden, D. (1986). On the role of the Obligatory Contour Principle in phonological theory. *Language*, 62(2):353–383.
- Odden, D. (1994). Adjacency parameters in phonology. *Language*, 70(2):289–330.
- Onn, F. M. (1980). *Aspects of Malay Phonology and Morphology: A Generative Approach*. Kuala Lumpur: Universiti Kebangsaan Malaysia.
- Rogers, J., Heinz, J., Bailey, G., Edlefsen, M., Visscher, M., Wellcome, D., and Wibel, S. (2010). On languages piecewise testable in the strict sense. In Ebert, C., Jäger, G., and Michaelis, J., editors, *The Mathematics of Language*, volume 6149 of *Lecture Notes in Artificial Intelligence*, pages 255–265. Springer.
- Schadeberg, T. (1979). Über die töne der verbalen formen im rimi. *Afrika und Übersee*, 57:288–313.

## References IV

- Sibanda, G. (2004). *Verbal phonology and morphology of Ndebele*. PhD thesis, UC Berkeley.
- Thomas, W. (1982). Classifying regular events in symbolic logic. *Journal of Computer and Systems Sciences*, 25:360—376.