Logic and the Generative Power of Autosegmental Phonology

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Introduction

- What is the computational complexity of nonlinear phonology?
- It appears that phonology is fundamentally *regular* or *sub-regular* (Johnson, 1972; Kaplan and Kay, 1994; Heinz and Idsardi, 2011)
- The *regular* classes of languages and relations are often defined from an *automata-theoretic* perspective; for example, regular languages are those definable with a finite-state acceptors

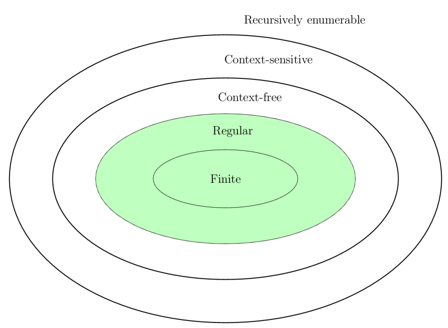
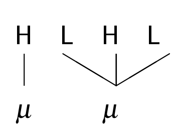


Figure 1: The Chomsky Hierarchy

- Automata-theoretic studies of segmental phonology argue that phonotactics and processes are finite-state (Heinz, 2007; Gainor et al., 2012; Chandlee and Heinz, 2012; Heinz and Lai, 2013)
- Attempts at viewing Autosegmental Phonology (Goldsmith, 1976, henceforth AP) from an automata-theoretic perspective are less clear: Wiebe (1992) argued that it is *necessarily* non-regular; Kornai (1995) offers a linearization he claims is finite-state, while Bird and Ellison (1994) argue otherwise, implementing AP with special 'synchronized' finite-state acceptors
- When looking at the complexity of AP from a MODEL-THEORETIC (MT) perspective, which builds models of strings using mathematical logic, it looks regular

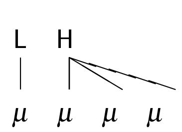
Autosegmental Diagrams

- The central insight of AP is that segments can interact via separate tiers, and association relation between tiers is not necessarily one-to-one (Goldsmith, 1976)
- For example, in (1), multiple positions on the tonal tier are associated to a single position on the segmental tier (here, assuming μ as the tonal-bearing unit)



(1)

- In (2), it's the other way around, with multiple positions on the segmental tier associated to one position on the tonal tier



(2)

- The following attempts to capture this kind of model in a MT framework to study its properties, like Graf (2010) does for Government Phonology

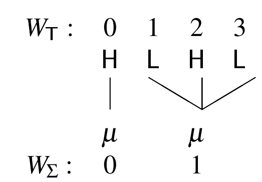
Word Models

- Each tier can be represented as a WORD MODEL $\mathscr{W} = \langle W, \lhd^*, P\rangle$:
  - W is the set of positions in the word
  - $\lhd^*$ is the precedence relation
  - P defines which positions are occupied by which symbols in the alphabet
- Word models for the tonal (T) and segmental (Σ) tiers in (1):

$$\begin{aligned}\mathscr{W}_{\mathsf{T}} = & \Big\langle \{0,1,2,3\}_{W_{\mathsf{T}}}, \{(0 \lhd^* 0), (0 \lhd^* 1), (0 \lhd^* 2), ..., (3 \lhd^* 3)\}_{\lhd^*}, \\ & \{0,2\}_{\mathsf{H}}, \{1,3\}_{\mathsf{L}} \Big\rangle \\ \mathscr{W}_{\Sigma} = & \Big\langle \{0,1\}_{W_{\Sigma}}, \{(0 \lhd^* 0), (0 \lhd^* 1), (1 \lhd^* 1)\}_{\lhd^*}, \{0,1\}_{\mu} \Big\rangle\end{aligned} \tag{3}$$

- Finally, we need an ASSOCIATION MODEL relating the two; $\mathscr{A} = \langle \mathscr{W}_{\mathsf{T}}, \mathscr{W}_{\Sigma}, \vartriangle\rangle$, where $\vartriangle$ is a relation relating positions in $\mathscr{W}_{\mathsf{T}}$ to positions in $\mathscr{W}_{\Sigma}$
- Association model for (1):

$$\mathscr{A} = \big\langle \mathscr{W}_{\mathsf{T}}, \mathscr{W}_{\Sigma}, \big\{(0 \vartriangle 0), (1 \vartriangle 1), (2 \vartriangle 1), (3 \vartriangle 1)\big\}_{\vartriangle}\big\rangle \tag{4}$$



(5)

- This is similar to Bird and Klein (1990)'s approach, except 1) precedence does not hold between tiers and 2) there is only one association relation (they have 'association' and 'overlap')

MSO

- We consider sets of models definable with MONADIC SECOND-ORDER LOGIC, which allows us to define strings with logical operators $\wedge, \vee, \neg$ (and thus $\rightarrow$), quantifiers $\forall$, and $\exists$, variables $(x, y, ...)$ and sets of variables $(X, Y, ...)$
- With these and $\lhd^*$, we can define other relations, such as immediate precedence $\lhd$ and less-than precedence $\lhd^+$ (see handout)
- Example: The constraint *HH (no consecutive Hs) can be defined as $(\forall x)\,[\mathsf{H}(x) \rightarrow (\forall y)\,[(x \lhd y) \rightarrow \neg \mathsf{H}(y)]]$
- Stringsets definable with MSO are exactly the regular languages (Büchi, 1960), so each separate tier will be regular
- Relations between strings definable in MSO are also regular (Benedikt et al., 2001)

Autosegmental Axioms

- The fundamental axioms of AP (Goldsmith, 1976) can thus be defined in MSO:
- **Axiom 1**. *The no-crossing constraint.*

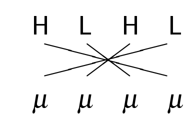
$$(\forall X, x, Y, y)\,\big[\big((X \vartriangle x) \wedge (Y \vartriangle y)\big) \rightarrow \big((X \lhd^+ Y) \leftrightarrow (x \lhd^+ y)\big)\big]$$

- **Axiom 2**. *One-to-one association (either left to right or right to left).*

$$(\forall X, x)\left[\begin{array}{l}((X \vartriangle x) \rightarrow (\forall y)\,[(y \lhd x) \rightarrow (\exists Y)\,[Y \vartriangle y]]) \vee \\ ((X \vartriangle x) \rightarrow (\forall y)\,[(x \lhd y) \rightarrow (\exists Y)\,[Y \vartriangle y]])\end{array}\right]$$

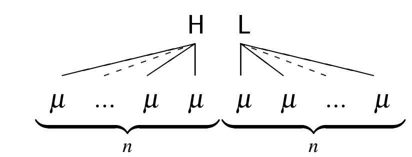
AP Axioms and Complexity

- Given the extra association relation $\vartriangle$, we cannot be guaranteed regularity
- However, Axioms 1 and 2 seem to conspire against non-regular representations
- **Axiom 1** bans a representation like (6), which is an autosegmental version of the non-regular relation $\{(w, w^R)\}$ (the set of strings matched with their reverse)



(6)

- **Axiom 2** bans a scenario like in (7) in which associations of two HL units spread out from the center of a segmental tier consisting of even numbers of μs; the yield of this would be the context-free stringset $\acute{\mu}^n \grave{\mu}^n$



(7)

- Conjecture: the linearization (yield) of any autosegmental representation that is at least restrained by Axioms 1 and 2 will be regular

Conclusions and Further Work

- AP *can* be captured with MSO, just as regular stringsets and relations
- Hypothesis: Any further language-specific restrictions can be further specified with MSO
- Can AP be captured with *first order* logic?
- We can explore what it is about tonal AP that makes it computationally different from segmental AP (Jardine, 2013)

References

Benedikt, M., Libkin, L., Schwentick, T., and Segoufin, L. (2001). A model-theoretic approach to regular string relations. In *Proceedings of the 16th Annual IEEE Symposium on Logic in Computer Science*.

Bird, S. and Ellison, T. M. (1994). One-level phonology: Autosegmental representations and rules as finite automata. *Computational Linguistics*, 20.

Bird, S. and Klein, E. (1990). Phonological events. *Journal of Linguistics*, 26:33–56.

Büchi, J. R. (1960). Weak second-order arithmetic and finite automata. *Zeitschrift für Mathematische Logik und Grundlagen der Mathmatik*, 6:66—92.

Chandlee, J. and Heinz, J. (2012). Bounded copying is subsequential: Implications for metathesis and reduplication. In *Proceedings of the 12th Meeting of the ACL Special Interest Group on Computational Morphology and Phonology*, pages 42–51, Montreal, Canada. Association for Computational Linguistics.

Gainor, B., Lai, R., and Heinz, J. (2012). Computational characterizations of vowel harmony patterns and pathologies. In *WCCFL*, pages 63–71.

Goldsmith, J. (1976). *Autosegmental Phonology*. PhD thesis, Massachussets Institute of Technology.

Graf, T. (2010). Logics of phonological reasoning. Master's thesis, University of California, Los Angeles.

Heinz, J. (2007). *The Inductive Learning of Phonotactic Patterns*. PhD thesis, University of California, Los Angeles.

Heinz, J. and Idsardi, W. (2011). Sentence and word complexity. *Science*, 333(6040):295–297.

Heinz, J. and Lai, R. (2013). Vowel harmony and subsequentiality. In Kornai, A. and Kuhlmann, M., editors, *Proceedings of the 13th Meeting on Mathematics of Language*, Sofia, Bulgaria.

Jardine, A. (2013). Computationally, tone is different. Under review.

Johnson, C. D. (1972). *Formal aspects of phonological description*. Mouton.

Kaplan, R. and Kay, M. (1994). Regular models of phonological rule systems. *Computational Linguistics*, 20:331–78.

Kornai, A. (1995). *Formal Phonology*. Garland Publication.

Wiebe, B. (1992). Modelling autosegmental phonology with multi-tape finite state transducers. Master's thesis, Simon Fraser University.