Markedness constraints are negative: An autosegmental constraint definition language

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1 Introduction

The content of markedness constraints is not arbitrary (Eisner 1997; de Lacy 2011; Rogers *et al.* 2013). Thus, any phonological theory should have some principled way of determining the set of possible markedness constraints. Such a theory of markedness constitutes our understanding of how humans decide well-formedness of phonological structure. What, then, is a *restrictive* yet *sufficient* theory of markedness constraints?

A theory of constraints can be specified by what de Lacy (2011) calls a *constraint definition language* (CDL). In this paper we argue for markedness CDL in which well-formedness constraints are fundamentally *negative*. This is based on independent principles of computational complexity and expressivity afforded by studying constraints as statements in formal logic.

We highlight some cases in autosegmental phonology in which it is difficult to write a negative constraint using the information which is usually explicitly included in autosegmental representations. We argue that it is undesirable to include *positive* constraints, as the requisite expressive power overgenerates. We then argue that it is preferable to enrich autosegmental representations by adding structure indicating when units are *not* associated. This is shown to approach the right level of expressivity without overgenerating to the same degree. The main lesson, then, is that a better theory enriches the structure in the representation rather than increasing the power of the formalism, because such a theory is *more restrictive*.

This paper is structured as follows. §2 establishes the context of the paper, which is developing a CDL capable of capturing well-formedness generalizations in natural language while excluding unattested constraints. §3 argues for mathematical logic as a CDL, and reviews the results of work applying logic over strings to phonological well-formedness. §4 shows how autosegmental representations pose some problems for the most restrictive of these logics. It then shows how it is more restrictive to solve these problems by enriching the autosegmental representations instead of increasing the power of the logical CDL. §5 concludes.

2 Background

2.1 Structural well-formedness

Speakers have knowledge regarding the phonotactic well-formedness of phonological structures. A famous example is Chomsky & Halle (1965)'s observation that English speakers prefer the nonword *blick* over **bnick*. This preference can be explained by the fact that **bnick* is not a well-formed structure given English phonotactics. Individual well-formedness generalizations can be formalized with *markedness constraints*. For example, that **bnick* is not well-formed in English can be attributed to its violation of Baertsch (2012)'s *TN constraint, which is violated when a nasal follows an obstruent in an onset.[1]

Markedness constraints can be split broadly into two categories: *negative* and *positive* constraints. *TN is a *negative* constraint; it bans a structure. Negative constraints are used quite often. Another famous example is the Obligatory Contour Principle (OCP; Leben 1973; McCarthy 1979), which is often invoked in autosegmental theories of phonology and thus relevant to this paper.

(1) The Obligatory Contour Principle (McCarthy 1986:p. 208)
At the melodic level, adjacent identical elements are prohibited.

The OCP is negative because it bans adjacent identical melodic elements. For example, (2b) contains a sequence of H tones on the tonal tier and thus violates (1). In contrast, (2a) does not violate (1) because it contains no such sequence.

(2) a. ✓ H
|
$\sigma\ \sigma\ \sigma$

b. * H H
| |
$\sigma\ \sigma\ \sigma$

Positive markedness constraints *requiring* structures are also commonly used in phonological analyses. One example is SPEC-T (Meyers 1997; Yip 2002), which requires that tone bearing units (TBUs) are specified for tones.

(3) SPEC-T: A TBU must be associated with a tone. (Yip 2002:83)

An example structure which satisfies this constraint is given below in (4a). In contrast, (4b) does not, because the second syllable is not specified for a tone.

(4) ✓ H L
| |
$\sigma\ \sigma$

* H
|
$\sigma\ \sigma$

SPEC-T is evaluated in a fundamentally different way than *TN and the OCP. The former looks for certain structures and is violated when it does not find them. The latter also look for certain structures, but are instead violated when they *do* find these structures. The purpose of this paper is to show how the latter is a more *restrictive* kind of constraint, and thus provides for a stronger theory of markedness. This is because generating the former kind of constraint requires a kind of computational power that also includes unattested markedness generalizations.

This paper is not concerned with the *interaction* of markedness constraints with faithfulness constraints (Prince & Smolensky 1993, 2004). Optimization over ranked faithfulness and markedness constraints is very expressive, even with simple constraints (Eisner 1997; Riggle 2004; Gerdemann & Hulden 2012). It is also not *necessary* to include faithfulness constraints, as it will be clear which theories of markedness produce unwanted well-formedness generalizations. This paper is thus concerned only with a *theory of markedness*, as the next section explains.

[1] It is also true that speakers can give gradient judgements; indeed, Chomsky & Halle (1965) found that English speakers preferred **bnick* over **bzick*, while judging *blick* to be better than both. As its focus is on the *expressivity* of constraints, this paper abstracts away from gradience. This does not lead to a loss of generality because any of the constraints here may be evaluated in either a categorical or gradient manner. For more discussion see (Heinz 2010).

2.2 Constraint definition languages

It is important to have a *theory* of what are possible well-formedness generalizations in human language phonology. For example, Eisner (1997) points out some logically possible, but theoretically undesirable, constraints, such as PALINDROMIC:

(5) PALINDROMIC: The candidate reads the same forwards and backwards (Eisner 1997:(1))

No language requires a constraint like PALINDROMIC, and so any sufficiently restrictive theory of phonology should somehow exclude it. In classical Optimality Theory (OT; Prince & Smolensky 1993, 2004), this is accomplished by stipulating that PALINDROMIC does not exist in the universal constraint set CON. However, this misses generalizations about similarities between constraints that *are* in CON that distinguish them from constraints like PALINDROMIC. Eisner (1997) argues that formalizing the idea of what constraints are *possible* brings such similarities to the fore. It also leads to a *falsifiable* theory of constraints, because it makes clear which constraints are expected to be impossible.

We can formalize a theory of constraints using what de Lacy (2011) calls a *constraint definition language* (CDL). A CDL needs to explicitly define the following:

(6) a. a set of possible constraints
b. how the constraints are interpreted

A CDL meeting the criteria in (6) is a restrictive, falsifiable theory of constraints argued for above. It is restrictive because it specifies the range of constraints, and possible constraints outside the CDL's range are hypothesized not to be attested.

Explicit CDLs have been offered before. Automata-theoretic CDLs have been given in Eisner (1997) and Riggle (2004). Potts & Pullum (2002) offer a CDL based in modal logic. The paper follows the work of Heinz (2010), Graf (2010), and Rogers *et al.* (2013), who compare logical CDLs within the framework of formal language theory (Hopcroft *et al.* 2006). As explained in the following section, these kinds of logical languages make strong candidates for CDLs, because not only do they meet the criteria in (6), but their expressivity is well-studied.

3 Logical constraint definition languages

3.1 A logical hierarchy

This section outlines how logical languages can be used as CDLs. As mentioned above, the work of Heinz (2010), Graf (2010), and Rogers *et al.* (2013) build on work relating formal language theory and formal logic and applying it to phonological theory. While this work is primarily concerned with strings, the discussion here lays the groundwork for §4, which brings up issues which come about when applying this work to autosegmental structures.

Logical languages make for excellent CDLs for several reasons. One, any logical language meets the criteria in (6) because it explicitly defines the range of constraints that can be written and the interpretation of those constraints (Potts & Pullum 2002). Two, there exists a well-studied *hierarchy* of logical languages in which some languages are more expressive than others. Taken as a CDL, each language in the hierarchy gives us a *hypothesis* about the range of constraints that should appear in phonology. We can thus compare these hypotheses to discover where phonological markedness lies on the logical hierarchy. As the relationship between logic and formal language theory is well-understood, these hypotheses also come with a cognitive interpretation based in computational complexity (Rogers *et al.* 2013). Finally, none of the logical languages here are capable of writing a constraint like PALINDROMIC (Büchi 1960).

Figure 1 gives a partial hierarchy of these logical languages. The vertical lines show inclusion relationships, with the higher languages properly including the lower ones. Thus, for example, any statement that can be made with a conjunction of negative literals can be made in propositional logic, but not vice versa. First-order logic is the most powerful logic in the hierarchy in Figure 1 (but not the most powerful logic possible).

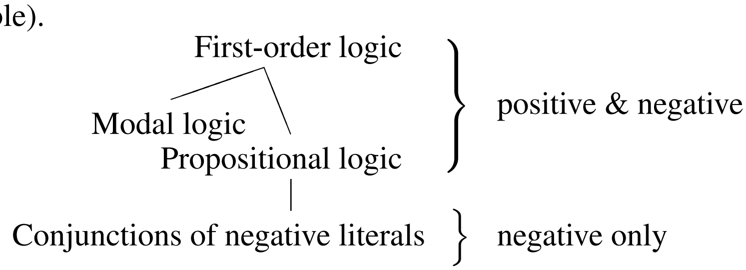


Figure 1: A hierarchy of logical languages.

Conjunctions of negative literals, which lie at the bottom of the hierarchy, are sufficient to describe many well-formedness generalizations in phonology (Heinz 2010; Rogers *et al.* 2013). This is significant, as negative literals are evaluated in a *local* manner by checking the substructures of a representation (Rogers *et al.* 2013). Of course, they are only capable of making negative statements, as indicated in Figure 1. Higher logics are capable of creating positive constraints, but this requires *global* evaluation, and thus is capable of creating bizarre constraints. Negative literals, then, provide a much stronger theory of markedness, although as we shall see, it is necessary to modify the structure in order to make them sufficiently expressive.

To illustrate, this paper will compare negative literals (NLs) with first-order (FO) logic. It will be clear that, although they are fundamentally negative, NLs provide a much better fit to the kind of markedness constraints seen in phonology. The arguments against FO in this paper also apply to propositional logic; FO was chosen because it is more straightforwardly applied to autosegmental constraints. Modal logic has also been applied to phonology (Potts & Pullum 2002; Graf 2010), however its relationship to negative literals is understudied. We thus leave it to future work, although we believe it is no more appropriate as a theory of markedness than NLs, especially considering non-linear structures (see Footnote 3).

We first review NLs and FO constraints with linear (i.e. string-like) structures. Given concerns of space, we only provide brief formal definitions of the logical languages. Readers interested in a more thorough treatment are referred to Rogers *et al.* (2013) or Heinz & Rogers (2014).

3.2 Negative literals

Consider the following very common well-formedness generalization.

(7) "Nasals must be voiced"

In any language for which this well-formedness generalization is true, a form like [pan] must be distinguished from from *[pan̥] (assuming [pan] violates no other well-formedness generalizations).

To define a logical language, we need to be explicit about the set of representations which the statements in the language will be about. In linear phonology (a la Chomsky & Halle 1968) representations are strings of feature bundles. The forms [pan] and *[pan̥] are given in (8) as strings of the features for nasality, voicing, and consonantal status (we omit the other features).

(8) a.

$$\begin{array}{ccc} \text{p} & \text{a} & \text{n} \\ \left[\begin{array}{c} -\text{nasal} \\ -\text{voice} \\ +\text{cons} \end{array}\right] & \left[\begin{array}{c} -\text{nasal} \\ +\text{voice} \\ -\text{cons} \end{array}\right] & \left[\begin{array}{c} +\text{nasal} \\ +\text{voice} \\ +\text{cons} \end{array}\right] \end{array}$$

b. *

$$\begin{array}{ccc} \text{p} & \text{a} & \text{n̥} \\ \left[\begin{array}{c} -\text{nasal} \\ -\text{voice} \\ +\text{cons} \end{array}\right] & \left[\begin{array}{c} -\text{nasal} \\ +\text{voice} \\ -\text{cons} \end{array}\right] & \left[\begin{array}{c} +\text{nasal} \\ -\text{voice} \\ +\text{cons} \end{array}\right] \end{array}$$

We can then define negative literals over these kind of structures. The definition of a negative literal is as follows.

(9) Definition of a negative literal
Given a set of structures A, a statement $\neg\phi$ is a *negative literal* if ϕ is a *substructure* of some $a \in A$. A structure a *satisfies* $\neg\phi$ if ϕ is not a substructure of a.

In essence, a negative literal is logical not ($\neg$) plus a substructure. Substructures are defined differently for different sets of structures—a substructure is any 'piece' of any valid structure in the set of structures we are interested in. As we are currently interested in strings of feature matrices, a substructure is thus any string of matrices containing some subset of the relevant features. For example, the following is a valid negative literal for the kind of representations exemplified in (8).

(10) $$\neg\left[\begin{array}{c} +\text{nasal} \\ -\text{voice} \end{array}\right]$$

The feature matrix [+nasal, −voice] is a valid substructure given Chomsky & Halle (1968)-style representations, as it is a substructure of any string of segments which contains a [+nasal, −voice] segment.

Again, as a NL, it fundamentally *negative*, as it is only satisfied by representations which do not contain this substructure. Thus, (8b) does not satisfy this constraint, as the segment [n̥] contains this particular set of features and values, as highlighted in bold below in (11).

(11) * $\begin{array}{ccc} \text{p} & \text{a} & \text{n̥} \\ \begin{bmatrix} -\text{nasal} \\ -\text{voice} \\ +\text{cons} \end{bmatrix} & \begin{bmatrix} -\text{nasal} \\ +\text{voice} \\ -\text{cons} \end{bmatrix} & \begin{bmatrix} \textbf{+nasal} \\ \textbf{−voice} \\ +\text{cons} \end{bmatrix} \end{array}$

In contrast, (8a) does not contain the substructure [+nasal, −voice], and so it satisfies (10). In this way, (10) distinguishes [pan] from *[pan̥] as (7) intends.

3.3 First-order logic

The constraint in (7) can also be written *positively* with a statement in FO. FO is a very different logic than NLs, as it includes variables which range over single positions in a representation. Variables are used to evaluate a finite set of *predicates* based on the properties of positions in a representation. Like NLs, the predicates available will differ according to the kind of representation. For the feature matrix representations we are using at the moment, we have the following predicates:

(12) a. For any feature $[\pm\text{F}]$, $[+\text{F}](x)$ and $[-\text{F}](x)$ are valid predicates. A predicate $[\alpha\text{F}](x)$ is evaluated to true if x is a $[\alpha\text{F}]$ segment.
b. $precedes(x, y)$ is a valid predicate. $precedes(x, y)$ is true iff x immediately precedes y.

FO statements are made up of these predicates and the standard boolean connectives ($\neg, \wedge, \vee, \rightarrow$) and whose variables are quantified by the standard quantifiers ($\exists, \forall$). For example, the following FO statement captures (7):

(13) $(\forall x)\big[[+\text{nasal}](x) \rightarrow [+\text{voiced}](x)\big]$

This statement reads "for all x, if x is nasal, then x is also voiceless." It is evaluated as follows. Each position (segment) in the representation is checked as x. If $[+\text{nasal}](x) \rightarrow [+\text{voiced}](x)$ is true for all positions x, then the statement is true for the entire representation.

We can thus see that (13) is true for (8a). For every segment in (8a), $[+\text{nasal}](x) \rightarrow [+\text{voiced}](x)$ is true, as the only segment which is [+nasal] is also [+voice]. In (8b), the [n̥] is [+nasal] but [−voice], and so $[+\text{nasal}](x) \rightarrow [+\text{voiced}](x)$ is false for [n̥]. Thus, (13) is false for (8b), and thus gets us exactly the distinction between (8a) and (b) that is intended by (7). It does this in a *positive* way, by requiring that any nasal x must also be voiced.

However, these kind of positive statements also allow bizarre constraints to be written in FO. For example, consider the following FO statement.

(14) $(\forall x, \exists y)\big[[+\text{nasal}](x) \rightarrow [-\text{voice}](y)\big]$

This constraint unpacks to the following generalization:

(15) "If there is a nasal, there must be a voiceless segment (somewhere in the word)"

The constraint in (14) makes the distinction between the following forms (feature matrices omitted):

(16) a. ✓[pan] b. ✓[pan̥] c. ✓[bad] d. *[ban]

According to (14), for all positions x, if x is [+nasal], there must be some position y which is [−voice]. This is true for (16a) through (c). The forms [pan] and [pan̥] have both [+nasal] segments ([n] and [n̥], respectively) as well as [−voice] segments ([p] or [n̥]), while [bad] has no [+nasal] segment, so it is not required to have a [−voice] segment. However, (16d) does not satisfy (14), because it has a voiced [n] but no voiceless segment.

To our knowledge, no natural language makes such a well-formedness distinction. Thus, as a theory of markedness, FO clearly *overgenerates*. This is due to the positive nature of FO. As the discussion following (13) explained, in order to interpret statements that require structures, the evaluation procedure must keep track of the truth value of each predicate for each position in the representation.

3.4 Interim conclusion: global versus local evaluation

In this way, the evaluation of FO constraints can be said to be *global*. This contrasts with NLs, which are fundamentally *local* because they can be evaluated by a procedure which simply checks each substructure of a certain size. Thus, it can be provably shown that it is impossible to write a constraint for the generalization in (15) using NLs. This is important, because like PALINDROMIC, constraints like (14) are unattested and should be excluded from our theory of markedness. Choosing NLs as a CDL accomplishes this.

It should be mentioned that NLs can generate some unnatural constraints. The following NL bans segments that are both [+nasal] and [+voice].

(17) $\neg\begin{bmatrix} +\text{nasal} \\ +\text{voice} \end{bmatrix}$

Arguably, the markedness generalization represented by (17) is unattested in natural language. However, this is because it is *phonetically* unnatural, not because it is somehow overly complex. The focus of this paper is to show that, in terms of complexity, NLs are preferable to FO. This is still true, as being strictly more powerful than NLs, FO can generate (17) as well as the other unattested examples discussed above. How we can further restrict NLs with phonetic knowledge is beyond the scope of this paper, but returned to again briefly in the conclusion.

Of course, while it is clear how NLs can capture local generalizations like the one in (7), it may be less clear how they may handle long-distance well-formedness generalizations such as consonant or vowel harmony (Hansson 2010; Nevins 2010). Heinz (2010) and Heinz *et al.* (2011) show how the same concept of evaluating substructures can capture long-distance generalizations. However, this requires adding structure to strings in order to capture distance relationships between segments.

This additional structure is preferable, though, to increasing the power of the logic—as discussed, higher levels of logic overgenerate in a way that NLs cannot. This a key lesson for dealing with the challenges of writing NL constraints for autosegmental representations.

4 Logical constraints and autosegmental structures

Rogers *et al.* (2013) discuss the logical hierarchy as applied to string structures. However, phonologists often use non-linear representations, the most widely used being *autosegmental phonology* (AP; Goldsmith 1976). It is thus yet an unstudied question how the logical hierarchy introduced in §3.1 can apply to AP representations (ARs). This is the focus of this paper. We choose AP over other non-linear alternatives (the Optimal Domains Theory of Cassimjee & Kisseberth 2001 or Q-theory of Shih & Inkelas 2014) because its formal properties are better understood (Bird & Klein 1990; Coleman & Local 1991; Goldsmith 1976), and the issues that arise in AP are likely to arise in other kinds of non-linear representations as well.

An AR is given below in (18). The key difference between ARs and strings are that elements in an AR are arranged into two (or more) separate strings called *tiers*. Elements on different tiers may be connected by *association lines*.

(18)
$$\begin{array}{cc} \mathrm{H} & \mathrm{L} \\ | & | \\ \sigma & \sigma \end{array}$$

As discussed above, when defining logical languages based on ARs, we must take this additional information into account.

This leads to two interesting complications when defining a negative markedness CDL using NLs over ARs. One is that analyses of phonological phenomena using ARs commonly use constraints which force TBUs (or some other kind of anchor unit) to be specified for a tone (or some other featural autosegment). This is a fundamentally positive constraint, as it *requires* an association. This is also the case for the other complication, in which spreading creates obligatory contour tones.

The following shows how these two kind of constraints cannot be captured by NLs over simple ARs which only include information about autosegments and the association lines between them. However, we also show how the arguments presented in the previous section against FO still hold for ARs. Thus, as increasing the power of the logic leads to overgeneration, we instead argue for enriching the structure of ARs. Specifically, we argue that for a restrictive markedness CDL, ARs need to include explicit information about when associations do *not* occur.

4.1 CNLs and nonlinear constraints

We first show some examples of how to extend the idea of NLs to AP. Recall that a negative literal is the negation of some substructure out of the set of possible representations. Since we are now considering the set of ARs as the set of possible representations, the possible literals have changed somewhat.

One, since the tiers of an AR are strings, we can use *substrings* (pieces of strings) of autosegments in our negative literals. For example, the OCP in (1) can be partially implemented with the NL in (19a).[2]

(19) a. NL: $\neg$HH b. ✓ $\begin{array}{ccc} \mathrm{H} & & \\ | & & \\ \sigma & \sigma & \sigma \end{array}$ c. * $\begin{array}{ccc} \mathbf{H} & \mathbf{H} & \mathrm{H} \\ | & | & | \\ \sigma & \sigma & \sigma \end{array}$

[2]To fully implement the OCP would require a conjunction such as $\neg$HH $\wedge$ $\neg$LL.

This distinguishes (19b) from (c), as HH is a substring of the tonal tier HHH in (c) but not of H in (b). One instance of this substring has been highlighted in (19c) in bold (in this and the following diagrams, for clarity we highlight only the initial instance of an included substructure).

Substructures in NLs can also include associations between autosegments. For example, in Hirosaki Japanese (Haraguchi 1977), H tones cannot be multiply associated. This constraint can be captured by the NL in (20a).

(20) a. NL: ¬ H
σ σ

b. ✓ H L
σ σ σ σ

c. * **H** L
σ σ σ σ

The NL in (20a) specifies a substructure in which a H tone is associated to two syllables. This is a substructure of (20c), and so this AR fails to satisfy the NL (whereas (20b) does satisfy it).

Thus, NLs can insightfully characterize some autosegmental constraints. However, consider a positive constraint like SPEC-T, originally given in (3). Recall that SPEC-T forces all TBUs to be specified for a tone. It distinguishes, for example, ARs like (21a) from (b).

(21) a. ✓ H L
σ σ

b. * H
σ σ

We cannot distinguish (21a) from (b) with a NL. The reason is that (21b) is a substructure of (21b)—note that (21a) is just (b) with an additional L tone and corresponding association. Thus, any substructure of (21b) is also a substructure of (a). For example, we cannot simply ban 'σ', as no AR which contains a syllable will satisfy such an NL.

(22) a. NL: $\neg\sigma$

b. * H L
σ σ

c. * H
σ σ

In general, any AR with underspecified TBUs is a substructure of an AR that is the same except those TBUs are specified. Thus, an NL can't ban the former without also banning the latter.

A similar situation occurs in cases in which spreading to create contours is obligatory. Consider Aghem (Hyman 2014), in which any H tone followed by an L spreads to the right, creating a falling contour:

(23) a. /é - nòm/ → [é - nôm] 'to be hot'
b. /fú - kìa/ → [fú - kîa] 'your sg. rat'
c. e-nom → e-nom [é - nôm] 'to be hot'
H L   H L

A constraint on surface well-formedness in Aghem making such spreading obligatory must then distinguish between the following two ARs.

(24) a. ✓ H L
σ σ

* H L
σ σ

In the well-formed (24a), the H has spread to create a contour, whereas in the ill-formed (24b), it has not.

These two cannot be distinguished with NLs, again because (24b) is a substructure of (24a). For example, the following NL in (25a) is satisfied by neither AR, as it is a substructure of both.

(25) a. NL: $\neg \begin{array}{cc} \mathrm{H} & \mathrm{L} \\ | & | \\ \sigma & \sigma \end{array}$ b. * $\begin{array}{cc} \mathbf{H} & \mathbf{L} \\ |\diagdown & | \\ \boldsymbol{\sigma} & \boldsymbol{\sigma} \end{array}$ c. * $\begin{array}{cc} \mathbf{H} & \mathbf{L} \\ | & | \\ \boldsymbol{\sigma} & \boldsymbol{\sigma} \end{array}$

These two cases establish a significant issue in creating a CDL for ARs using NLs. Unlike with string representations, we may need to mark as ill-formed ARs which are substructures of ARs which we want to mark as well-formed.

There are two ways in which we can respond to this issue. One is to increase the power of the CDL, for example by moving from NLs to FO. The other is to enrich the structure by adding abstract elements to our ARs. The remainder of the paper argues for the latter choice. In §4.2, we show that FO statements over ARs are also capable of generating undesirable constraints just as they did for strings in §3.3. In contrast, §4.3 shows how we can get to the right level of expressiveness by adding information to ARs that has already been used in the literature.

4.2 FO and nonlinear constraints

This section shows how FO over ARs succumbs to the same problems of overgeneration as FO over strings. First, we need to define a set of basic predicates for use with ARs. Now, variables x, y, etc., range over autosegments in AR diagrams.

(26) a. For any S out of the possible autosegments, S(x) is a valid predicate. A predicate S(x) is evaluated to true if x is an autosegment S. For example, H(x) is true when x is a H tone autosegment.

b. $x = y$ is a valid predicate. $x = y$ is true when x and y are the same autosegment. Similarly, let $x \neq y$ be true when x and y are distinct autosegments.

c. $precedes(x, y)$ is a valid predicate. $precedes(x, y)$ is true when x immediately precedes y.

d. $assoc(x, y)$ is a valid predicate. $assoc(x, y)$ is true when x and y are associated.

e. $assoc$-H(x) is a valid predicate. $assoc$-H(x) is true when $(\exists y)[assoc(x, y) \wedge \mathrm{H}(y)]$ is true. Similarly for $assoc$-L(x).

Let FO statements over ARs be built out of the predicates (26) and the same quantifiers and logical connectives introduced in §3.3.

With FO statements, we can easily capture the positive AR constraints which were shown to be problematic for NLs in §4.1. We start with SPEC-T. Written in FO, SPEC-T translates to the FO statement in (27).

(27) $(\forall x)\big[\sigma(x) \rightarrow (assoc\text{-H}(x) \vee assoc\text{-L}(x))\big]$

The statement in (27) specifies that for all x, if x is a syllable, it must either be associated to a H tone or a L tone. This distinguishes the AR in (21a) from (b), as repeated below in (28).

(28) a. ✓ $\begin{array}{cc} \text{H} & \text{L} \\ | & | \\ \sigma & \sigma \end{array}$ b. * $\begin{array}{cc} \text{H} & \\ | & \\ \sigma & \boldsymbol{\sigma} \end{array}$

In (28a), each position satisfies $\sigma(x) \rightarrow (assoc\text{-H}(x) \vee assoc\text{-L}(x))$, because every autosegment in (28a) is either not a σ or is a σ associated to some tone. In (28b), however, the second syllable (highlighted in bold) fails this statement, because it is not associated to any tone. Thus, (28b) does not satisfy (27). In this way, we can write a positive constraint in FO for SPEC-T.

Similarly, we can write an FO statement for the spreading constraint from Aghem. Recall that in Aghem a H must spread to a following L-toned syllable. This behavior can be specified by the FO statement in (29).

(29) $$(\forall x, y, z)\big[\big(precedes(x, y) \wedge \text{H}(z) \wedge assoc(x, z) \wedge assoc\text{-L}(y)\big) \rightarrow assoc(z, y)\big]$$

The statement in (29) requires that for any triplet x, y, z of autosegments in an AR, if x and y precede each other, and x is associated to a H autosegment z, and y is associated to some L, then z must also be associated to y. This constraint captures the right distinction for Aghem, as shown below in (30).[3]

(30) a. ✓ $\begin{array}{cc} \text{H} & \text{L} \\ | \diagdown & | \\ \sigma & \sigma \end{array}$ b. * $\begin{array}{cc} \textbf{H} & \text{L} \\ | & | \\ \boldsymbol{\sigma} & \boldsymbol{\sigma} \end{array}$

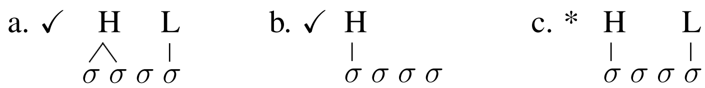
The statement in (29) fails for (30b) when x is the first syllable, y is the second syllable, and z is the H. The offending triplet is highlighted in bold in (30b). The first syllable (x) precedes the second (y), the first syllable is associated to a H (z), and the second syllable is associated to a L. Thus, the antecedent of the implication in (29) is true for this x, y, z. However, the consequent is not, as the H (z) is not also associated to the second syllable (y). Thus the statement is not true for this x, y, z, and so (29) fails for (30b). However, the reader can confirm that for any triplet of autosegments in (30a) the statement in (29) is true. Thus, we can distinguish (30a) from (b) and capture the generalization in Aghem.

Thus, FO logic over ARs are sufficiently powerful to create the constraints that NLs could not. However, we do not want to posit FO logic as a CDL because, as before, FO can generate bizarre constraints. For instance, consider (31).

(31) $$(\forall w, \exists x, y, z)\big[\text{L}(w) \rightarrow \text{H}(x) \wedge assoc(x, y) \wedge assoc(x, z) \wedge y \neq z\big]$$

This statement captures the generalization "If there is an L, there must also be a doubly associated H". For example, it distinguishes the following ARs:

[3]This distinction is beyond the power of modal logic. Briefly, modal logic is equivalent to FO restricted to two variables (Graf 2010), but (29) clearly needs to refer to three variables.

(32) 

As far as we are aware, no language makes such distinctions. Another such constraint is given in (33).

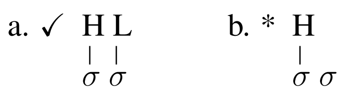
(33) $$(\exists x, y, z)\big[assoc\text{-H}(x) \wedge assoc\text{-H}(y) \wedge assoc\text{-H}(y) \wedge x \neq y \wedge x \neq z \wedge y \neq z\big]$$

This statement specifies that in every APR, there must be *exactly* 3 distinct TBUs specified for a H tone. Again, we are not aware of any such constraint in a language. However, if FO is our CDL, then (33) is a valid markedness constraint.

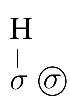
This section has shown that, just as for strings, FO overgenerates as a CDL. This is because FO computes globally over the entire AR structure. The next section shows how we can keep our constraints *local* by adding some information to ARs.

4.3 Enriching the structure

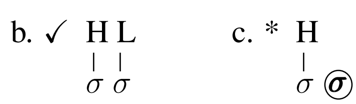
We can capture constraints requiring associations with NLs if we enrich the structure to make explicit when associations are absent. Recall from §4.1 that the problem with these constraints is that we need to ban some ARs which happen to be subgraphs of other ARs that we want to keep. We saw one example when trying to implement SPEC-T, repeated below in (34).

(34) 

Again, with NLs we cannot keep (34a) to the exclusion of (b), because (b) is a subgraph of (a). However, this problem can be addressed with the addition of an explicit contrast between specified and unspecified syllables. Borrowing notation from Pulleyblank (1986), we can require that all unspecified syllables are marked as ⓢ. Thus (34b) would not be a valid AR, and instead (35) would be.

(35) 

With σ autosegments introduced, we can posit NLs banning them. This gets us exactly the distinction we need, as seen in (36).

(36) a. NL: $\neg\sigma$ 

The NL in (36a) bans (36c) without banning (b). This is because, with the contrast between σ and σ, (36c) is no longer a substructure of (b). While this is additional structural information, it has been used before in AP rule formalisms.

The Aghem case requires a slightly different kind of information. However, this information has, at least implicitly, been used in past AP analyses. Recall that spreading in Aghem requires the following distinction in well-formedness, which is impossible for NLs because (37b) is a substructure of (a).

(37) a. ✓ H L
σ σ

b. * H L
σ σ

In (37b), the H did *not* spread when it could have. We can make this explicit with *antiassociation* lines denoting the *absence* of association. Thus, (37b) would instead be as in (38), where antiassociation lines are denoted by dotted lines.

(38) H L
σ σ

We can then formulate the constraint is Aghem with the following NL banning a structure in which a H does not spread to a following L-toned syllable:

(39) a. NC: ¬ H L
σ σ

b.✓ H L
σ σ

c. * **H L**
$\boldsymbol{\sigma}$ $\boldsymbol{\sigma}$

The constraint in (39a) distinguishes between (39b), in which the H does spread to the L-toned syllable, and (39c), in which it doesn't. Again, this is possible because (39c) is not a substructure of (39b), as (b) has an association line between the H and the second syllable where (c) has an antiassociation line between them.

Are explicit antiassociation lines novel? Actually, they are not as strange as they seem at first blush. First, antiassociation lines mark a *potential* association not realized, just like ⓢ. Second, knowledge of the absence of association has been crucial for past analyses of phenomena using AP. For example, take the following constraint from Walker (2011, 2014)'s recent analyses of vowel harmony:

(40) ∀HARMONY
For every feature F in a word, a violation is assigned to every vowel to which F is not associated.

Any implementation of ∀HARMONY must have some way of detecting associations that didn't occur. This is exactly what antiassociation lines are.

This section has shown some concrete examples of how additional structure can be used to increase the expressive power of a CDL based in NLs. Of course the principles by which this information is added remain to be explored. In particular, antiassociation lines raise a few questions. For example, do they observe the No-Crossing Constraint (Coleman & Local 1991; Hammond 1988)? Walker's constraint suggests that they might not, as it implies antiassociation lines from each feature to each vowel in the word. Unfortunately, a detailed discussion of a *Representation* Definition Language (RDL) including antiassociation lines is beyond the scope of this paper. However, regardless of the shape such an RDL would take, we know that it will lead to a more restrictive CDL, as it is provably impossible to get constraints like the FO examples with NLs.

5 Conclusion

In conclusion, negative constraints are extremely restrictive, while allowing positive constraints overgenerates. Thus, although evaluating well-formedness over autosegmental structures with negative constraints requires additional structure, this is preferable to including positive constraints. In this way, we have seen independent motivation for explicitly including Pulleyblank (1986)'s unspecified TBU notation and failed associations a la Walker (2011, 2014)'s $\forall$HARMONY constraint. Finally, this paper's conclusions show the importance of a Representation Definition Language. Fleshing out a full theory is beyond the scope of this paper, but we hope we have provided a starting point for future work. Such work can also examine how this RDL can incorporate phonetic naturalness, a la Hayes *et al.* (2004) or Sebastian & Heinz (2015), to complement the ideas of complexity focused on here.

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