Boolean Monadic Recursive Schemes for Phonological Analysis: A tutorial

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Overview

- A theory of phonology...
  - allows us to directly state linguistically significant generalizations;
  - captures abstract universals about the phonological cognitive module;
  - (and is also learnable)

Overview

- **Boolean monadic recursive schemes (BMRS)** is a logical formalism for implementing such a theory

Overview

An example:

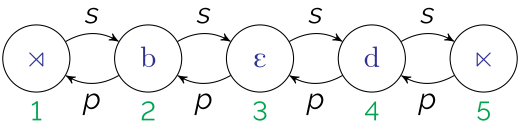
$$\acute{\sigma}_{\mathrm{o}}(x) \;=\; \texttt{if}\ \mathsf{final}_{\mathrm{i}}(x)\ \texttt{then}\ \bot\ \texttt{else}\ \texttt{if}\ \acute{\sigma}_{\mathrm{o}}(p(x))\ \texttt{then}\ \top\ \texttt{else}\ \acute{\sigma}_{\mathrm{i}}(x)$$

in:	σ	$\acute{\sigma}$	σ	σ	σ	σ
$\acute{\sigma}_{\mathrm{i}}(x)$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$	$\bot$
$\acute{\sigma}_{\mathrm{o}}(x)$	$\bot$	$\top$	$\top$	$\top$	$\top$	$\bot$
out:	σ	$\acute{\sigma}$	$\acute{\sigma}$	$\acute{\sigma}$	$\acute{\sigma}$	σ

Boolean Monadic Recursive Schemes (BMRS)

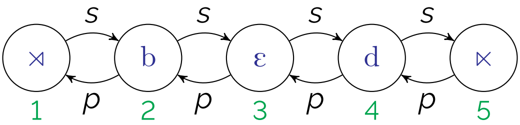
- A logical characterization of a phonological process includes:
  1. **Models** (representations)
  2. a **logical language** for describing properties
  3. an **interpretation** for describing the *output* structure

BMRS: String Models



- **indices** (elements in the structure)
- **order functions** p and s
- **properties** of the indices

BMRS: String Models



- **Featural properties:**

	1	2	3	4	5
$[\text{sonorant}]_{\mathrm{i}}(x)$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$
$[\text{voice}]_{\mathrm{i}}(x)$	$\bot$	$\top$	$\top$	$\top$	$\bot$
$\ltimes_{\mathrm{i}}(x)$	$\bot$	$\bot$	$\bot$	$\bot$	$\top$
$\ltimes_{\mathrm{i}}(s(x))$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$

$\top$ = true
$\bot$ = false

BMRS: Logical language

The core of BMRS syntax are expressions of the form

$$\texttt{if } A \texttt{ then } B \texttt{ else } C$$

that return boolean values ($\top$ or $\bot$)

BMRS: Logical language

Ex.,

$$\texttt{if}\ [\text{son}]_{\mathrm{i}}(x)\ \texttt{then}\ \bot\ \texttt{else}\ [\text{voi}]_{\mathrm{i}}(x)$$

BMRS: Logical language

Ex.,

$$\texttt{if } [\text{son}]_\text{i}(x) \texttt{ then } \bot \texttt{ else } [\text{voi}]_\text{i}(x)$$

	⋊	b	ɛ	d	⋉
$[\text{sonorant}]_\text{i}(x)$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$
$[\text{voice}]_\text{i}(x)$	$\bot$	$\top$	$\top$	$\top$	$\bot$
	$\bot$	$\top$	$\bot$	$\top$	$\bot$

BMRS: Logical language

A, *B*, or *C* can be another expression

$$\texttt{if } [\text{son}]_{\text{i}}(x) \texttt{ then } \bot \texttt{ else}$$
$$\texttt{if } [\text{voi}]_{\text{i}}(x) \texttt{ then } \top \texttt{ else } \bot$$

Usually this is *C*, to chain together expressions

BMRS: Logical language

Expressions define new properties

$$\begin{bmatrix} -\text{son} \\ +\text{voi} \end{bmatrix}_{\text{i}}(x) := \texttt{if } [\text{son}]_{\text{i}}(x) \texttt{ then } \bot \texttt{ else } [\text{voi}]_{\text{i}}(x)$$

BMRS: Logical language

Expressions define new properties

$$\begin{bmatrix} -\text{son} \\ +\text{voi} \end{bmatrix}_{\mathrm{i}}(x) := \texttt{if } [\text{son}]_{\mathrm{i}}(x) \texttt{ then } \bot \texttt{ else } [\text{voi}]_{\mathrm{i}}(x)$$

$$\mathrm{final}_{\mathrm{i}}(x) := \texttt{if } \ltimes (s(x)) \texttt{ then } \top \texttt{ else } \bot$$

BMRS: Logical language

Expressions define new properties

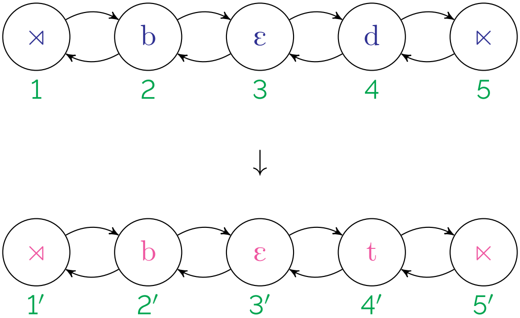
$$\begin{bmatrix} -\text{son} \\ +\text{voi} \end{bmatrix}_{\text{i}}(x) := \texttt{if } [\text{son}]_{\text{i}}(x) \texttt{ then } \bot \texttt{ else } [\text{voi}]_{\text{i}}(x)$$

$$\text{final}_{\text{i}}(x) := \texttt{if } \ltimes (s(x)) \texttt{ then } \top \texttt{ else } \bot$$

$$\text{D\#}_{\text{i}}(x) := \texttt{if } \begin{bmatrix} -\text{son} \\ +\text{voi} \end{bmatrix}_{\text{i}}(x) \texttt{ then } \text{final}(x) \texttt{ else } \bot$$

BMRS: Interpretations

Word-final obstruent devoicing:



BMRS: Interpretations

	$\rtimes$	b	ɛ	d	$\ltimes$
$[\text{sonorant}]_i(x)$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$
$[\text{voice}]_i(x)$	$\bot$	$\top$	$\top$	$\top$	$\bot$
$[\text{cor}]_i(x)$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$
$[\text{sonorant}]_o(x)$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$
$[\text{voice}]_o(x)$	$\bot$	$\top$	$\top$	$\bot$	$\bot$
$[\text{cor}]_o(x)$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$
	$\rtimes$	b	ɛ	t	$\ltimes$

- **interpretations** specify maps by defining output structures in terms of the input structures (Engelfriet & Hoogeboom 2001)

BMRS: Interpretations

Scheme - series of definitions of (output) properties

$$
\begin{aligned}
[\text{son}]_{\text{o}}(x) &= \ldots \\
[\text{voi}]_{\text{o}}(x) &= \ldots \\
[\text{cor}]_{\text{o}}(x) &= \ldots
\end{aligned}
$$

Properties in a **BMRS** are

- **boolean**
- **monadic** (unary)
- **recursive**

BMRS: Interpretations

Output properties assert the conditions under which a segment is $+$ for a given feature in the output structure.

$$\begin{aligned} [\text{son}]_o(x) &= \ldots \\ [\text{voi}]_o(x) &= \ldots \\ [\text{cor}]_o(x) &= \ldots \end{aligned}$$

BMRS: Interpretations

Word-final obstruent devoicing:

$$\begin{aligned}
[\text{son}]_{\text{o}}(x) &= \ldots \\
[\text{voi}]_{\text{o}}(x) &= \ldots \\
[\text{cor}]_{\text{o}}(x) &= \ldots
\end{aligned}$$

	$\rtimes$	b	ɛ	d	$\ltimes$
$[\text{son}]_{\text{i}}(x)$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$
$[\text{voi}]_{\text{i}}(x)$	$\bot$	$\top$	$\top$	$\top$	$\bot$
$[\text{cor}]_{\text{i}}(x)$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$
$[\text{son}]_{\text{o}}(x)$					
$[\text{voi}]_{\text{o}}(x)$					
$[\text{cor}]_{\text{o}}(x)$					
	$\rtimes$	b	ɛ	t	$\ltimes$

BMRS: Interpretations

Word-final obstruent devoicing:

$$\begin{aligned} [\text{son}]_o(x) &= [\text{son}]_i(x) \\ [\text{voi}]_o(x) &= \ldots \\ [\text{cor}]_o(x) &= \ldots \end{aligned}$$

	⋊	b	ɛ	d	⋉
$[\text{son}]_i(x)$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$
$[\text{voi}]_i(x)$	$\bot$	$\top$	$\top$	$\top$	$\bot$
$[\text{cor}]_i(x)$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$
$[\text{son}]_o(x)$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$
$[\text{voi}]_o(x)$					
$[\text{cor}]_o(x)$					
	⋊	b	ɛ	t	⋉

BMRS: Interpretations

Word-final obstruent devoicing:

$$
\begin{aligned}
[\text{son}]_o(x) &= [\text{son}]_i(x) \\
[\text{voi}]_o(x) &= \ldots \\
[\text{cor}]_o(x) &= [\text{cor}]_i(x)
\end{aligned}
$$

	⋊	b	ɛ	d	⋉
$[\text{son}]_i(x)$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$
$[\text{voi}]_i(x)$	$\bot$	$\top$	$\top$	$\top$	$\bot$
$[\text{cor}]_i(x)$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$
$[\text{son}]_o(x)$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$
$[\text{voi}]_o(x)$					
$[\text{cor}]_o(x)$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$
	⋊	b	ɛ	t	⋉

BMRS: Interpretations

Word-final obstruent devoicing:

$$\begin{aligned}
[\text{son}]_\mathrm{o}(x) &= [\text{son}]_\mathrm{i}(x) \\
[\text{voi}]_\mathrm{o}(x) &= \texttt{if } \text{D\#}_\mathrm{i}(x) \texttt{ then } \bot \texttt{ else } [\text{voi}]_\mathrm{i}(x) \\
[\text{cor}]_\mathrm{o}(x) &= [\text{cor}]_\mathrm{i}(x)
\end{aligned}$$

	⋊	b	ɛ	d	⋉
$[\text{son}]_\mathrm{i}(x)$	⊥	⊥	⊤	⊥	⊥
$[\text{voi}]_\mathrm{i}(x)$	⊥	⊤	⊤	⊤	⊥
$[\text{cor}]_\mathrm{i}(x)$	⊥	⊥	⊥	⊤	⊥
$[\text{son}]_\mathrm{o}(x)$	⊥	⊥	⊤	⊥	⊥
$[\text{voi}]_\mathrm{o}(x)$	⊥	⊤	⊤	⊥	⊥
$[\text{cor}]_\mathrm{o}(x)$	⊥	⊥	⊥	⊤	⊥
	⋊	b	ɛ	t	⋉

BMRS: Recursion

H-tone spread to penult

$$\begin{array}{lcl}
\acute{\sigma}\sigma\sigma & \mapsto & \acute{\sigma}\acute{\sigma}\sigma \\
\sigma\acute{\sigma}\sigma\sigma\sigma\sigma & \mapsto & \sigma\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\sigma \\
\sigma\sigma\acute{\sigma}\sigma\sigma\sigma & \mapsto & \sigma\sigma\acute{\sigma}\acute{\sigma}\acute{\sigma}\sigma \\
\sigma\acute{\sigma}\sigma\sigma\sigma\sigma\sigma\sigma & \mapsto & \sigma\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\sigma
\end{array}$$

etc.

BMRS: Recursion

H-tone spread to penult

$$\begin{array}{lcl} \acute{\sigma}\sigma\sigma & \mapsto & \acute{\sigma}\acute{\sigma}\sigma \\ \sigma\acute{\sigma}\sigma\sigma\sigma\sigma & \mapsto & \sigma\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\sigma \\ \sigma\sigma\acute{\sigma}\sigma\sigma\sigma & \mapsto & \sigma\sigma\acute{\sigma}\acute{\sigma}\acute{\sigma}\sigma \\ \sigma\acute{\sigma}\sigma\sigma\sigma\sigma\sigma\sigma & \mapsto & \sigma\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\sigma \\ \text{etc.} & & \end{array}$$

$$\acute{\sigma}_{\mathrm{o}}(x) = ?$$

BMRS: Recursion

H-tone spread to penult

$$\sigma\acute{\sigma}\sigma\sigma\sigma\sigma \quad \mapsto \quad \sigma\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\sigma$$

in:	σ	$\acute{\sigma}$	σ	σ	σ	σ
$\acute{\sigma}_{\mathrm{i}}(x)$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$	$\bot$
$\acute{\sigma}_{\mathrm{o}}(x)$						
out:	σ	$\acute{\sigma}$	$\acute{\sigma}$	$\acute{\sigma}$	$\acute{\sigma}$	σ

$\acute{\sigma}_{\mathrm{o}}(x) =$

BMRS: Recursion

H-tone spread to penult

$$\sigma\acute{\sigma}\sigma\sigma\sigma\sigma \quad \mapsto \quad \sigma\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\sigma$$

in:	σ	$\acute{\sigma}$	σ	σ	σ	σ
$\acute{\sigma}_{\mathrm{i}}(x)$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$	$\bot$
$\acute{\sigma}_{\mathrm{o}}(x)$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$	$\bot$
out:	σ	$\acute{\sigma}$	$\acute{\sigma}$	$\acute{\sigma}$	$\acute{\sigma}$	σ

$\acute{\sigma}_{\mathrm{o}}(x) =$

$\acute{\sigma}_{\mathrm{i}}(x)$

BMRS: Recursion

H-tone spread to penult

$$\sigma\acute{\sigma}\sigma\sigma\sigma\sigma \quad \mapsto \quad \sigma\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\sigma$$

in:	σ	$\acute{\sigma}$	σ	σ	σ	σ
$\acute{\sigma}_{\mathrm{i}}(x)$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$	$\bot$
$\acute{\sigma}_{\mathrm{o}}(x)$	$\bot$	$\top$	$\top$	$\top$	$\top$	$\top$
out:	σ	$\acute{\sigma}$	$\acute{\sigma}$	$\acute{\sigma}$	$\acute{\sigma}$	σ

$$\acute{\sigma}_{\mathrm{o}}(x) = \texttt{if } \acute{\sigma}_{\mathrm{o}}(p(x)) \texttt{ then } \top \texttt{ else } \acute{\sigma}_{\mathrm{i}}(x)$$

BMRS: Recursion

H-tone spread to penult

$$\sigma\acute{\sigma}\sigma\sigma\sigma\sigma \quad \mapsto \quad \sigma\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\sigma$$

in:	σ	$\acute{\sigma}$	σ	σ	σ	σ
$\acute{\sigma}_{\mathrm{i}}(x)$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$	$\bot$
$\acute{\sigma}_{\mathrm{o}}(x)$	$\bot$	$\top$	$\top$	$\top$	$\top$	$\bot$
out:	σ	$\acute{\sigma}$	$\acute{\sigma}$	$\acute{\sigma}$	$\acute{\sigma}$	σ

$$\acute{\sigma}_{\mathrm{o}}(x) = \begin{array}{l} \texttt{if}\ \mathrm{final}(x)\ \texttt{then}\ \bot\ \texttt{else} \\ \texttt{if}\ \acute{\sigma}_{\mathrm{o}}(p(x))\ \texttt{then}\ \top\ \texttt{else} \\ \acute{\sigma}_{\mathrm{i}}(x) \end{array}$$

BMRS: Review

BMRSs are

- logical descriptions of maps
- series of definitions of the form

$$[\text{F}]_{\text{o}}(x) = \begin{array}{l} \texttt{if (condition 1)(x) then } \top/\bot \texttt{ else} \\ \texttt{if (condition 2)(x) then } \top/\bot \texttt{ else} \\ \vdots \\ [\text{F}]_{\text{i}}(x) \end{array}$$

- computationally restrictive (Bhaskar et al., 2020)

A Homework Assignment: Iny

- Iny (Ribeiro 2002, 2012) ATR harmony requires both reference to input and output

/r-ɛ-rɔ=r-e/	[rerore]	"I ate it"
/b-Ø-ɪ-krɔ=kre/	[bikrokre]	"You will cut it"
/r-ɛ-hãɗɛ=r-e/	[rɛhãɗere]	"I hit it'
/brɔrɛ-dĩ/	[broreni]	"cow"
/wa-θɛ-riʧɔrɛ/	[waθeriʧɔrɛ]	"my sibling"
/kɔɗʊ-dĩ/	[kɔɗuni]	"a type of turtle" PL'
/r-ɛ-hɪ=r-e/	[rɛhire]	"I drove it away"

Iny in BMRSs

- Some hints:
  1. Define the relevant sets of natural classes,
  2. then write a formula to define the conditions under which vowels surface as [$\pm$ATR] (and the other features).

Iny in BMRSs

1. Natural class properties
   - $[+\text{ATR}, +\text{hi}]_{\text{i}}(x) =$
   - $[+\text{ATR}, -\text{hi}, -\text{lo}, -\text{nas}]_{\text{o}}(x) =$

2. Output features
   - $[\text{high}]_{\text{o}}(x) =$
   - $[\text{low}]_{\text{o}}(x) =$
   - $[\text{nasal}]_{\text{o}}(x) =$
   - $[\text{ATR}]_{\text{o}}(x) =$

Iny in BMRSs

Answers on

`adamjardine.net/bmrstutorial`

Ideas for AMP 2023 Submissions on BMRS

- What restrictions should we put on BMRS for defining natural classes?
- What does a tertiary feature system look like in BMRS? See Turkish voicing alternations as described in e.g., Inkelas (1995).
- BMRS captures elsewhere condition-type effects well. What about non-derived environment blocking?
- What is the status of intermediate representations? See, e.g., Gleim (2019) for a feeding Duke of York analysis of tone-epenthesis interactions in Arapaho.
- How does BMRS capture the typology of stress patterns? (E.g., in Gordon 2002)